

元智大學 102 學年度研究所 碩士班 招生試題卷

系(所)別: 機械工程學系碩士班 組別: 乙組 科目: 工程數學 用紙第 1 頁共 2 頁

●不可使用電子計算機

1. Using the method of variation of parameters to solve the differential equation. (16 %)

$$y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1, \quad x > 0$$

2. Using the method of Laplace Transformation to solve the initial value problem of $y(t)$. (17 %)

$$y'' + 4y' + 3y = e^t \quad \text{with} \quad y(0) = 0, \quad \left. \frac{dy}{dt} \right|_{t=0} = 2$$

3. Given a curve $C: \vec{r}(t) = \cosh t \vec{i} + \sinh t \vec{j}$. (13 %)

(a) Find a tangent vector $\vec{r}'(t)$ and the corresponding unit tangent vector $\vec{u}(t)$. (5 %)

(b) Find $\vec{r}'(t)$ and $\vec{u}(t)$ at the given point $P: \left(\frac{5}{3}, \frac{4}{3}, 0\right)$. (5 %)

(c) Find the tangent at P . (3 %)

4. What kind of conic section (or pair of straight lines) is given by the quadratic form

$$4x_1x_2 + 3x_2^2 = 1? \quad \text{Transform it to principal axes. Express } \mathbf{x}^T = [x_1 \quad x_2] \text{ in terms of the new}$$

coordinate vector $\mathbf{y}^T = [y_1 \quad y_2]$. (10 %)

5. Using Green's theorem, evaluate the line integral $\oint_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counterclockwise around the

boundary C of the region R , where $\vec{F} = \left[\frac{e^y}{x}, e^y \ln x + 2x \right]$, $R: 1 + x^4 \leq y \leq 2$. (10 %)

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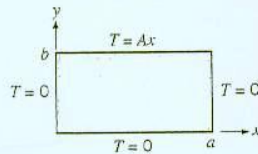
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用紙第 2 頁共 2 頁

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6. A two-dimensional rectangular plate is subjected to the boundary conditions shown as below. Derive an expression for the steady state temperature distributions $T(x,y)$ with solving the heat conduction equation. (17%)



The heat conduction equation is :

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0$$

Please find the solution in sin, cos, sinh, cosh series functions by the method of separation variables.

7. There is a partial differential equation of one dimensional wave equation to modeling vibrating string. The equation, boundary and initial conditions are following.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, u(0,t) = 0, u(l,t) = 0 \text{ for all time.}$$

The initial profile of the string is ,

$$f(x) = \begin{cases} \frac{2k}{l}x, & \text{when } 0 < x < l/2, \\ \frac{2k}{l}(l-x), & \text{when } l/2 < x < l \end{cases}$$

initial velocity is zero, $g(x) = 0$.

Please find the solution in sin, cos series functions by the method of separation variables. (17%)

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