元智大學 102 學年度研究所 **碩士班** 招生試題卷

系(所)別：機械工程學系碩士班　組別：丙組　科目：工程數學　用紙第 1 頁共 2 頁

●不可使用電子計算機

1. Using the method of variation of parameters to solve the differential equation. (16 %)

$$y'' - \frac{4}{x}y' + \frac{4}{x^2}y = x^2 + 1 \quad , \quad x > 0$$

2. Using the method of Laplace Transformation to solve the initial value problem of $y(t)$. (17 %)

$$y'' + 4y' + 3y = e^t \quad \text{with} \quad y(0) = 0 \quad , \quad \left.\frac{dy}{dt}\right|_{t=0} = 2$$

3. Given a curve C: $\vec{r}(t) = \cosh t\,\vec{i} + \sinh t\,\vec{j}$. (13 %)

   (a) Find a tangent vector $\vec{r}'(t)$ and the corresponding unit tangent vector $\vec{u}(t)$. (5 %)

   (b) Find $\vec{r}'(t)$ and $\vec{u}(t)$ at the given point $P:\left(\frac{5}{3}, \frac{4}{3}, 0\right)$. (5 %)

   (c) Find the tangent at P. (3 %)

4. What kind of conic section (or pair of straight lines) is given by the quadratic form $4x_1x_2 + 3x_2^2 = 1$? Transform it to principal axes. Express $\mathbf{x}^T = [x_1 \quad x_2]$ in terms of the new coordinate vector $\mathbf{y}^T = [y_1 \quad y_2]$. (10 %)

5. Using Green's theorem, evaluate the line integral $\oint_C \vec{F}(\vec{r}) \bullet d\vec{r}$ counterclockwise around the boundary C of the region R, where $\vec{F} = \left[\frac{e^y}{x}, e^y \ln x + 2x\right]$, $R: 1 + x^4 \le y \le 2$. (10 %)

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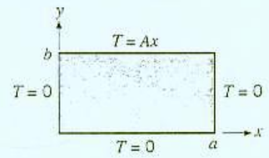
6. A two-dimensional rectangular plate is subjected to the boundary conditions shown as below. Derive an expression for the steady state temperature distributions T(x,y) with solving the heat conduction equation. (17 %)



The heat conduction equation is :

$$\frac{\partial^2 T(x,y)}{\partial x^2}+\frac{\partial^2 T(x,y)}{\partial y^2}=0$$

Please find the solution in sin, cos, sinh, cosh series functions by the method of separation variables.

7. There is a partial differential equation of one dimensional wave equation to modeling vibrating string. The equation, boundary and initial conditions are following.

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}\text{, u(0,t) = 0, u(l,t) = 0 for all time.}$$

The initial profile of the string is ,

$$f(x)=\begin{cases}\frac{2k}{l}x, \text{when } 0<x<i/2, \\ \frac{2k}{l}(l-x), \text{when}\cdots i/2<x<l\end{cases}$$

initial velocity is zero, g(x) = 0.

Please find the solution in sin, cos series functions by the method of separation variables. (17 %)

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