元智大學 102 學年度研究所 **碩士班** 招生試題卷

系(所)別：工業工程與管理學系碩士班　組別：不分組　科目：作業研究　用紙第 1 頁共 2 頁

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1. (30%) Consider the following linear programming problem.

Maximize $Z = X1 - X2 + 2X3$
Subject to

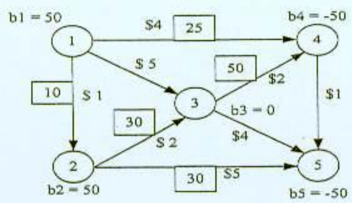
$$3X1 - 2X2 + 2X3 \le 22$$
$$X1 - X2 + X3 \le 8$$
$$-X1 + 2X2 + 2X3 \le 10$$
$$X1, X2, X3 \ge 0$$

Answer the following questions.

(a) Find the basic variables with respect to the solution, (X1, X2, X3) = (2, 0, 6), when the Simplex method is applied (using X4, X5, X6 as slack variables for the three inequality constraints). What is the basic feasible solution? (5%)
(b) Write down the basis corresponding to this basic feasible solution. (5%)
(c) Check if this basic feasible solution is optimal? (5%)
(d) Compute the dual optimal feasible solution. (5%)
(e) What is the range of profit C1 (currently C1 = 1) such that the current optimal solution remains unchanged? (5%)
(f) What is the range of resource 3 (currently b3 = 10) such that the optimal basis is not changed? (5%)

2. (25%) Consider the following capacitated minimum cost flow problem, where nodes 1and 2 each can supply 50 units, node 3 is a transshipment station (b3 =0), and nodes 4 and 5 each requires 50 units. The numbers within squares are transshipment capacities; for example, the arc from node 1 to node 4 (arc 1→4) can transport at most 25 units. The number after the dollar sign ($) on each arc represents the transportation cost per unit of goods. For example, the transportation cost per unit on arc 1→4 is 4 dollars. Answer the following questions.

(a) Write down the LP formulation for this problem. (5%)
(b) Start with the following initial solution: $X_{12} = 10$, $X_{14} = 25$, $X_{13} = 15$, $X_{23} = 30$, $X_{25} = 30$, $X_{34} = 25$, $X_{35} = 20$, and apply the network simplex method to find the optimal BFS to this problem. (10%)
(c) What is the range of C_{45} (currently $1) so that the current optimal BFS remains optimal? (5%)
(d) Suppose that the supply of node 1 increases from 50 units to 60 units, and node 5 also increases its demand from 50 to 60. What is the new optimal solution? (5%)



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3. (15%) Apply dynamic programming technique to solve the following integer nonlinear programming problem.

$$\text{Max } Z = 18X_1 - X_1^2 + 20X_2 + 10X_3$$

$$\text{Subject to} \quad 2X_1 + 4X_2 + 3X_3 \le 11$$

$$X_1, X_2, X_3 \ge 0 \text{ and are integers.}$$

4. (15%) Consider two M/G/1/∞ queueing models, A and B. The two models have the same Poisson arrival process with a mean rate $\lambda = 5$ per hour, but their service time distributions are different. The service times of Model A are i.i.d. normally distributed with a mean time $1/\mu_A = 2/15$ per hour and a standard deviation $\sigma_A = 1/5$ hour. The service times of Model B are a hyper-exponential distribution; i.e., with probability $p = 0.5$, the distribution is exponential with a mean rate $\mu_1 = 5$ per hour, and with probability $1 - p = 0.5$, the distribution is exponential with a mean rate $\mu_2 = 10$ per hour. The expected waiting time in the queue (L_q) of a customer in the steady state is as follows:

$$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}, \text{ where } \rho = \frac{\lambda}{\mu}$$

Answer the following questions:
(a) What is the fraction of time that Model A is empty (no customers in the system)? (3%)
(b) What is the fraction of time that Model B is empty? (3%)
(c) What is the expected number of customers in the system (L) for Model A? (3%)
(d) What is the expected number of customers in the system (L) for Model B? (3%)
(e) Which model has longer expected waiting time in the system (W)? (3%)

5. (15%) Consider the following Markov chain with transition matrix given as follows.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1/3 & 2/3 & 0 & 0 & 0 \\ 1/2 & 1/4 & 1/4 & 0 & 0 \\ 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 3/4 & 0 & 0 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(a) List a set of equations that can compute the expected number of transitions from state 1 to the absorbing state 5? Do not compute. (10%)
(b) List a set of equations for computing the following conditional probability: Pr{Chain visits state 3 before entering state 5 | currently the chain is at state 4}. Do not compute. (5%)

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