

# 元智大學 102 學年度研究所 碩士班 招生試題卷

系(所)別：工業工程與管理 組別：不分組 科目：機率與統計 用紙第 1 頁共 2 頁  
學系碩士班

●不可使用電子計算機

1. (20 points)

- (a) State the "Central Limit Theorem" in detail. (8 points)
- (b) State the "Chebyshev's Inequality" in detail. (6 points)
- (c) State the "Markov's Inequality" in detail. (6 points)

2. (20 points) At an electronics plant, it is known from past experience that the probability is 0.85 that a new worker who has attended the company's training program will meet the production quota and that the corresponding probability is 0.35 for a new worker who has not attended the company's training program. Also, 80% of all new workers attend the training program.

- (a) What is the probability that a new worker will meet the production quota? (10 points)
- (b) Find the probability that a new worker who meets the production quota will have attended the company's training program. (10 points)

3. (20 points) If \$ $x$  is invested in mutual fund A, its worth after 1 year,  $X_A$ , is normally distributed with mean  $1.05x$  and variance  $0.0004x^2$ , i.e.,  $X_A \sim N(1.05x, 0.0004x^2)$ . And if \$ $x$  is invested in mutual fund B, its worth after 1 year,  $X_B$ , is normally distributed with mean  $1.05x$  and variance  $0.0009x^2$ , i.e.,  $X_B \sim N(1.05x, 0.0009x^2)$ . Suppose that you have \$5000 to invest and that you place \$ $y$  in mutual fund A and \$(5000 -  $y$ ) in mutual fund B.

- (a) What is the expect value of the total worth of your investment after 1 year? (5 points)
- (b) What is the variance of the total worth of your investment after 1 year? (5 points)
- (c) What value of  $y$  minimizes the variance of the total worth of your investment after 1 year? (5 points)
- (d) If you adopt this "conservative" strategy, what is the probability that after 1 year the total worth of your investment is more than 5300? (Express your answer in terms of  $\Phi(a) = P(Z \leq a)$  where  $Z$  follows the standard normal distribution and  $a > 0$ .) (5 points)

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4. (20 points) An investigator wishes to estimate the proportion of students at a certain university who have violated the honor code. Having obtained a random sample of  $n$  students, she realizes that asking each, "Have you violated the honor code?" will probably result in some untruthful responses. Consider the following scheme, the investigator makes a total of 100 balls in a box with 50 RED balls and 50 BLUE balls. RED ball, then the question is "Have you violated the honor code (yes or no)?" BLUE ball, then the question is "Is the last digit of your cell phone number a 0, 1, or 2 (yes or no)?"
- Each student in the random sample is asked to pick one ball randomly from the box, and answer the resulting question truthfully. Because of the irrelevant question on BLUE balls, a yes response no longer stigmatizes the respondent, so we assume that responses are truthful. Let  $p$  denote the proportion of honor-code violators (i.e., the probability of a randomly selected student being a violator), and let  $\beta = P(\text{yes response})$ . Let  $Y$  denote the number of yes response.
- (a) Derive an unbiased estimator for  $p$  based on  $Y$ . (12 points)
- (b) Referring to question (a), derive a 95% upper confidence bound for  $p$ , assume  $n$  is large. (8 points)

5. (20 points) An experimenter is interested in the hypothesis testing problem

$$H_0: \mu \leq 3.00\text{mm} \quad \text{versus} \quad H_1: \mu > 3.00\text{mm}$$

where  $\mu$  is the average thickness of a set of glass sheets. Suppose that a sample of  $n = 9$  glass sheets is obtained and their thicknesses are measured and that the experimenter wishes to use a value of  $\sigma = 0.06$ .

- (a) For what values of the  $z$ -statistic does the experimenter accept the null hypotheses with a level  $\alpha = 0.1$ ? (5 points)
- (b) Suppose that the sample mean is 3.02mm, is the null hypothesis accepted or rejected with level  $\alpha = 0.05$ . (5 points)
- (c) Calculate the exact  $p$ -value, if the sample mean is 3.02mm. (Express your answer in terms of  $\Phi(a) = P(Z \leq a)$  where  $Z$  follows the standard normal distribution and  $a > 0$ .) (5 points)
- (d) Explain the meaning of  $p$ -value. (5 points)

For your reference,  $z_{0.025} = 1.96$ ,  $z_{0.05} = 1.645$ ,  $z_{0.1} = 1.28$

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