

# 淡江大學102學年度碩士班招生考試試題

系別: 數學學系

科目: 機率與統計

考試日期: 3月10日 (星期日) 第3節

本試題共 6 大題, 1 頁

注意事項: (1) 請按題號順序作答。(2) 可用鉛筆。(3) 不可使用計算機。(4) 交回答案卷、題目卷、計算紙。

1. (10分) Let  $X$  be a random variable (r.v.) that takes values from 0 to 9 with equal probability. Find the pmf of the random variable  $Y = X \bmod(3)$ , where  $a \bmod(m)$  denotes the remainder when  $a$  is divided by  $m$ .
2. (20分) Let the random variables  $X$  and  $Y$  have a joint pdf which is uniform over the rectangle with vertices at  $(0, 0)$ ,  $(0, 1)$  and  $(1, 0)$ .
  - (a) Find the conditional pdf of  $X$  given  $Y$ .
  - (b) Find  $E[X|Y = y]$ .
  - (c) Find the value of  $E[X]$ .
3. (20分) Let  $X_1, X_2, \dots$  be independent random variables that are uniformly distributed over  $[-1, 1]$ .
  - (a) Let  $Y_n = \prod_{i=1}^n X_i$ , show that the sequence  $Y_1, Y_2, \dots$  converges in probability to 0.
  - (b) Let  $Y_n = \max_{1 \leq i \leq n} \{X_i\}$ , show that the sequence  $Y_1, Y_2, \dots$  converges in probability to 1.
4. (22分) Suppose we have a random sample  $X_1, \dots, X_n$  from a shifted exponential distribution, that is,  $X_i = \delta + Y_i$ , where  $Y_1, \dots, Y_n$  are a random sample from an  $Exp(1)$  distribution and  $\delta > 0$  is the unknown parameter. Let  $x_1, x_2, \dots, x_n$  be a dataset that is a realization of a random sample  $X_1, \dots, X_n$ .
  - (a) Draw the likelihood  $L(\delta; x_1, \dots, x_n)$  and determine the MLE for  $\delta$ .
  - (b) Let  $T = \min\{X_1, \dots, X_n\}$ , find the cdf of  $T$ .
  - (c) Find the  $100(1 - \alpha)\%$  confidence interval for  $\delta$ .
5. (14分) A number  $x$  was generated from a uniform distribution on the interval  $[0, \theta]$ . One decides to test  $H_0: \theta = 2$  against  $H_1: \theta \neq 2$  by rejecting  $H_0$  if  $x \leq 0.1$  or  $x \geq 1.9$ .
  - (a) Compute the probability of committing a type I error.
  - (b) Compute the probability of committing a type II error if the true value of  $\theta$  is 2.5.
6. (14分) Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution with pdf  $f(x; \theta) = \theta(1 - x)^{\theta-1}$ ,  $0 < x < 1$ , zero elsewhere, where  $\theta > 0$ .
  - (a) Find the form of the uniformly most powerful test of  $H_0: \theta = 1$  against  $H_1: \theta > 1$ .
  - (b) What is the likelihood ratio test statistic for testing  $H_0: \theta = 1$  against  $H_1: \theta \neq 1$ .