淡江大學 102 學年度碩士班招生考試試題

系別：數學學系　　　　　　　　科目：微積分（含高微）

考試日期：3 月 10 日(星期日) 第 2 節　　　　　本試題共 10 大題， 1 頁

1. (12 points) Find the following limits:

   (1) $\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$　　(2) $\lim_{x\to(\frac{\pi}{2})^-}(\tan x)^{\cos x}$

2. (10 points) Find an equation of the tangent to the curve $y=\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$ at the point (0,-1).

3. (10 points) Let f(x) = x^3+2x. Show that f has inverse and find $\frac{df^{-1}}{dx}$ at x=3.

4. (8 points) Find dy/dx if y = $\int_0^{x^2}\frac{1}{2+3\sqrt{t}}dt$.

5. (10 points) Find $\partial z/\partial x$ if z is defined implicitly as a function of x and y by the equation $x^3+y^3+z^3$ +6xyz = 1.

6. (10 points) Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty}(\frac{1}{4})^n n\frac{x^n}{(n^2+1)}$.

7. (10 points) Evaluate the integral $\int_0^3\int_{\sqrt{\frac{x}{3}}}^1 e^{y^3}$ dydx.

8. (10 points) Show that f(x) = $\sqrt{x}$ is uniformly continuous on [0, ∞).

9. (10 points) Show that the sequence {nxe^{-nx^2}} converges uniformly to 0 on [a, ∞) for every a > 0.

10. (10 points) Prove that if $\{a_n\}$ is monotone increasing and bounded, then $\{a_n\}$ is convergent.