淡江大學 102 學年度碩士班招生考試試題

系別：國企系、產經系、管科系　　　科目：微積分

考試日期：3 月 10 日(星期日) 第 3 節　　　　本試題共　7　大題，　1　頁

1. Evaluate the limit if it exists. (18%)

   (1) $\lim\limits_{t \to 0}\left(\dfrac{1}{t} - \dfrac{1}{t^2 + t}\right)$　　　　(2) $\lim\limits_{n \to \infty}\left(1 + \dfrac{2}{n}\right)^n$　　　　(3) $\lim\limits_{x \to 0}\left(\dfrac{x}{e^x - 1}\right)$

2. The unemployment rate $U(t)$ varies with time. The table (from the Bureau of Labor Statistics) gives the percentage of unemployed in the US labor force from 1999 to 2008. (12%)

t	$U(t)$	t	$U(t)$
1999	4.2	2004	5.5
2000	4.0	2005	5.1
2001	4.7	2006	4.6
2002	5.8	2007	4.6
2003	6.0	2008	5.8

   (a) What is the meaning of $U'(t)$?

   (b) Estimate the value of $U'(2003)$.

3. A company estimates that the number of units x of a new product, measured in thousands, that can be sold and the price p, in dollars, of each unit are related by the equation $px^2 + 15px = 30000$. Find dx/dp when the price is \$30 per unit. (10%)

4. Let $R(x) = e^{-x^2}$.
   (a) Find the intervals of concavity and inflection points. (10%)
   (b) Sketch the graph of $R(x)$. (10%)

5. Find the absolute maximum and minimum values of the function
   $f(x) = x^3 - 3x^2 + 1$ as $-2 \le x \le 4$. (10%)

6. Evaluate the following integrals. (18%)

   (1) $\int x^2 \ln x\, dx$　　　　(2) $\int x^2 \sqrt{1 + x}\, dx$　　　　(3) $\int \dfrac{1 + x^2 + x^4}{x^4}\, dx$

7. Find the maximum and minimum values of the function
   $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$. (12%)