

科目：微積分

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一. 計算下列各題: (64 %)

(a) 
$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^{2n} (\ln(n+k) - \ln n)$$

(b) 
$$\lim_{x \rightarrow \infty} x e^{-x^2} \int_1^x e^{t^2} dt$$

(c) Find the derivative of the function  $f(x) = \frac{[x^3]}{2+x^2}$  whenever it exists. ( $[x^3]$  stands for the greatest integer less than or equal to  $x^3$ )

(d) 
$$\int_1^{\infty} \frac{\sqrt{x}}{(1+x)^2} dx$$

(e) 
$$\int_{-\infty}^{\infty} (\cos x) e^{-x^2/2} dx$$

(f) 
$$\sum_{n=2}^{\infty} \frac{n(n-1)}{5^n}$$

(g) 
$$\iint_R e^{(y-x)/(y+x)} dx dy$$
, where  $R$  is the region in the  $xy$ -plane bounded by trapezoid with vertices  $(0, 1)$ ,  $(0, 2)$ ,  $(2, 0)$ , and  $(1, 0)$ .

(h) 
$$\int_0^2 \int_{y^3}^8 y^2 e^{x^2} dx dy$$

二. The demand function for a certain make of ink-jet cartridge is  $p = -0.01x^2 - 0.1x + 6$ , where  $p$  is the unit price in dollars and  $x$  is the quantity demanded each week, measured in units of a thousand. Compute the elasticity of demand and determine whether the demand is inelastic, unitary, or elastic when  $x = 10$ . (12 %)

三. Suppose an investment is expected to generate income at the rate of  $f(t) = 30000 + 800t$  dollars/year for the next 5 years. Find the present value of this investment if the prevailing interest rate is 5%/year compounded continuously. (12 %)

四. (a) Find the Maclaurin series for  $e^{-\frac{1}{10}x^2}$ . (b) Approximate  $\int_0^1 e^{-\frac{1}{10}x^2} dx$  to four decimal places. (12 %)

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。