

科目：微積分

系所組：統計資訊學系應用統計碩士班

1. (20 %) Find the following limits

(a)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$

(b)  $\lim_{n \rightarrow \infty} (1 + \frac{3}{n})^{4n}$

(c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$

(d)  $\lim_{n \rightarrow \infty} \frac{1}{n^4} \sum_{i=1}^n i^3$

2. (30 %) Calculate the integrals

(a)  $\int x^3 e^{x^4} dx$

(b)  $\int e^x \sin x dx$

(c)  $\int_0^1 x^5 (1-x)^7 dx$

(d)  $\int_0^\infty x^{-1/2} e^{-x} dx$

(e)  $\int_0^6 [x] dx$  ([ ] is the greatest integer function)

3. (10 %) Find all relative extrema of  $f(x, y) = x^2 + y^3 - 6x - 12y$  and identify any saddle points.

4. (10 %) Solve the differential equation and initial condition:  
 $y' + 3y = 12e^x$  and  $y(0) = 5$ .

5. (10 %) Find the volume bounded by the surface  $f(x, y) = \sin y^2$  and the planes  $x = 0$ ,  $y = x$  and  $y = 1$ .

6. (20 %)

(a) Approximate  $e^{0.1}$  using the third Taylor polynomial for  $e^x$ , and estimate the error.

(b) Approximate  $\int_0^1 e^{-x^2} dx$  to three decimal places.

(c) Find  $\frac{d}{dx} \int_x^{x^2} e^{t^2} dt$ .

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。