

科目：微積分

系所組：企業管理學系管理學碩士班乙組

共本試題共 10 大題，每大題 10 分。請依下列規則作答：

1. 請務必註明題號，未在答案卷作答者，不予計分。
2. 請在答案卷書寫各題計算過程與最終答案，無計算過程者，不予計分。

1A. Find the derivative of $\sin^4(4x^3 + 1)$

1B. Find the derivative of $\ln(e^{10x}) + \ln(e^{5x} + 5) + 2e^{x^5}$

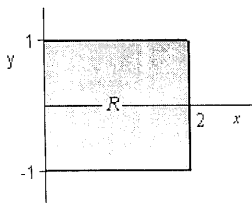
2. Evaluate the integral $\int \frac{3 \sin t}{\cos t + 5} dt$

3. Evaluate the integral $\int \frac{16x+64}{e^{4x}} dx$

4. Evaluate the integral $\int_0^{\ln 4} 3xe^x dx$

5. A company sells smart phones. The monthly sales of one type of smart phone, S , are predicted to be $S(x) = x^2(300 - 2x^2)$ thousand, in which x is the months after the product launch. Find the rate of change of the sales after 6 months in thousands per month.

6. The population density (people per square km) x km east and y km north of the center of a metropolitan city is $P(x, y) = 20000e^{x-y}$. Find the total population of the region R shown in the diagram below.



※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

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7. Suppose the birth rate in a country has decreased from $20e^{0.04t}$ million births per year to $20e^{0.02t}$ million births per year since 1975, where t is the number of years since 1975. Find the total decrease in population that would result from this lower birth rate between 1975 ($t = 0$) and 2000 ($t = 25$).

8. Consider the circle $(x - R)^2 + y^2 = r^2$, with $0 < r < R$. If we rotate this circle 360 degrees around the y axis to get a torus. Determine the interior volume of this torus.

9. Solve the differential equation and initial condition $\begin{cases} y' + 5xy = 10x \\ y(0) = 0 \end{cases}$

10. According to Newton's Law of Cooling, the rate of change in temperature of a cooling body is proportional to the difference between the temperature of the body and the surrounding environmental temperature. Therefore, the Newton's Law of Cooling can be expressed as the differential equation,

$$\frac{dT}{dt} = -k(T(t) - T_e), \text{ with } T(0) = T_0$$

$T(t)$ is the temperature of the body after t minutes of cooling, T_e is the environmental temperature, T_0 is the initial temperature of the object, and k is a parameter. Suppose we cool a cup of hot coffee, $T_0 = 85^\circ\text{C}$, at an environment of $T_e = 25^\circ\text{C}$. After 10 minutes, the coffee has cooled to a temperature of 65°C . Determine the value of k .

【Note: $\ln 3 = 1.0986$; $\ln 2 = 0.6931$ 】

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