(102)輔仁大學碩士班招生考試試題	考試日期：102 年 03 月 08 日 第　節
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科目： 軟體設計　　**系所組：** 資訊工程研究所

1. **(10 points)**
   (a) Please explain the sentence "Time complexity of an algorithm K is $O(n^2)$". **(Note:** "n" represents the total data size. **) (5 points)**
   (b) Please explain the notation "$2n^2 + \theta(n) = \theta(n^2)$". **(5 points)**

2. **(10 points)**
   (a) Show that the summation $\sum_{i=0}^{n} \lceil \log_2 i \rceil$ is $O(n\log n)$. **(5 points)**
   (b) Given a set $A=\{a_1, a_2, \ldots, a_n\}$ of n integers, describe in pseudo-code, an efficient method for computing each of average value $v_k = \sum_{i=1}^{k} a_i / k$, for k = 1,2,…n. What is the running time of this method? **(5 points)**

3. **(10 points)** Please give a recursive method that finds the minimum and maximum values in an array of **int** values without using any loops.

4. **(10 points)** Please explain the algorithm design method "divide and conquer". Use "Merge Sort" as an example to explain how to analyze the time- and space-complexity of "Merge Sort" which is designed by the "divide and conquer" method.

5. **(10 points)** Please explain the algorithm design method "dynamic programming". Given a list $A=\{a_1, a_2, \ldots, a_n\}$ of n integers, design an algorithm to find the longest ascending subsequence of the list A. Take a list A = {1,7,2, 5, 6,3} for example. The longest ascending subsequence of the list A is {1,2,5,6}. Additionally, analyze the time- and space-complexity of your designed algorithm.

6. **(10 points) Sorting Algorithm:** Fill in the blanks with "always", "sometimes", or "never".

   Sorting algorithm is very important. The time complexity of sorting algorithm is ___①___ O(n) in average case. A bubble sort of an array of 30 elements will ___②___ take 29 passes. Through quick sort, sorting an array of 30 elements will ___③___ require 29 partitions. Each partition ___④___ has at least one element in its final sorted position. Insertion sort is ___⑤___ more efficient than quick-sort in average. The time complexity of quick sort is ___⑥___ O(nlogn). Heap sort is ___⑦___ a stable sorting algorithm and its time complexity is ___⑧___ O(nlogn). Additionally, binary search tree can also be used for handling sorting. If a node x in a binary search tree has two children, then the successor of node x ___⑨___ has one left child. Searching a key in a binary search tree ___⑩___ takes O(logn) time. (Note: "n" represents the total data size.)

7. **(10 points)** Given a priority queue Q, answer the following two problems.
   (a) Can max (min) heap be used to implement Q? Please explain what max (min) heap is and justify your answer. **(5 points)**
   (b) Both sorted linked list and unsorted linked list can be used to implement Q. Please compare the time complexity of deleting data from the priority queue Q using these two linked lists. **(5 points)**

8. **(10 points)**
   (a) For a tree T, let n_I denote the number of its internal nodes, and let n_E denote the number of its external nodes. Show that if every internal nodes in T has exactly 4 children, then $n_E = 3n_I + 1$. **(5 points)**
   (b) Draw a binary tree T such that its preorder traversal is ABDICEGFH and its inorder traversal is DIBAGECFH. **(5 points)**

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

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9. **(10 points) Graph Theory:** Fill in the blanks with "always", "sometimes", or "never".

A spanning tree of graph G is an acyclic subgraph of G which connects all of the vertices in G. If G is a connected undirected graph, applying DFS (Depth-First Search) algorithm to G ___①___ produces tree and back edges. Spanning trees of a graph G has ___②___ the same number of edges. Any two spanning trees of a graph G ___③___ have a common edge. Additionally, minimum spanning tree problem is the most important issue we concerned. Minimum spanning tree of a graph G is ___④___ unique. The weight of minimum spanning tree of a graph G is ___⑤___ unique. Minimum spanning tree found by the Prim's algorithm is ___⑥___ unique. Except minimum spanning tree, shortest path problem in a graph is also a very popular issue. The path length from a vertex u to a vertex v in a minimum spanning tree of graph G is ___⑦___ a shortest path from vertex u to vertex v in the graph G. Additionally, if G is a directed graph with negative edges, then the answer of a single source shortest path problem solved by the Dijkstra's algorithm is ___⑧___ wrong and solved by the "Bellman Ford algorithm" is ___⑨___ wrong. Furthermore, if G is a directed graph with a negative cycle, then there is ___⑩___ a pair of vertices that has a shortest path.

10. **(10 points)** Consider a directed graph *G*.
   (a) Can DFS (Depth-First Search) be used to find a cycle in G? You have to justify your answer. No points will be given if you don't give the justification. **(5 points)**
   (b) Can BFS (Breadth-First Search) be used to check whether G is connected? You have to justify your answer. No points will be given if you don't give the justification. **(5 points)**