(102)輔仁大學碩士班招生考試試題	考試日期：102 年 3 月 8 日第 2 節
	本試題共 2 頁 (本頁為第 1 頁)

科目： 數學　　系所組： 資訊工程學系

1. Let $A = \{2, 3, 4, 9, 12, 18\}$, and define a binary relation R on set A by
xRy if $x|y$ (i.e., x divides y).
(a) Draw the Hasse diagram for the partially ordered set (A, R). (5%)
(b) Is the partially ordered set (A, R) a lattice? You need to explain your answer. (5%)

2. Consider the grammar $G = (N, T, P, S)$, where $N = \{S, A\}$, $T = \{a, b\}$, and $P = \{S \to aA, A \to Sb|b\}$.
(a) Is G a type-3 (regular) grammar? (3%)
(b) Is G a type-2 (context-free) grammar? (3%)
(c) Describe (in set-theoretic notation) the language generated by G. (4%)

3.
(a) Let $f(x)$ be the generating function for the sequence $a_0, a_1, a_2, a_3, \cdots$.
Prove that $\frac{1}{1-x}f(x)$ is the generating function for the sequence
$a_0, a_0 + a_1, a_0 + a_1 + a_2, a_0 + a_1 + a_2 + a_3, \cdots$ (5%)
(b) Find the generating function (in closed form) for the sequence $a_0, a_1, a_2, \cdots$, where $a_n = \sum_{i=0}^{n} \frac{1}{i!}$. (5%)

4. If $< H, \circ >$ and $< K, \circ >$ are subgroups of $< G, \circ >$, prove or disprove each of the following statements.
(a) $< H \cap K, \circ >$ is also a subgroup of $< G, \circ >$. (5%)
(b) $< H \cup K, \circ >$ is also a subgroup of $< G, \circ >$. (5%)

5. Consider an algebra $< A, \circ >$, where A is the set of all functions from set X to X, and $\circ$ denotes the usual composition of functions. If $f \in A$ and f has an inverse, prove that f is bijective (i.e., one to one and onto). (10%)

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

6. Solve the following system: (10%)

$$\begin{cases} x_1 - 3x_2 - 2x_3 = -5 \\ x_2 - x_3 = 4 \\ -2x_1 + 3x_2 + 7x_3 = -2 \end{cases}$$

7. Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find all eigenvalues and all eigenvectors of A. (10%)

8. Brief answers. (30%)

(a) Determine if $\mathbf{w} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}$ is in the null space of **A**, where $\mathbf{A} = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}$.
Explain your answer.

(b) Let T: $\mathbf{R}^2 \to \mathbf{R}^2$ with $T(\mathbf{x}) = \mathbf{Ax}$. What matrix has the effect of reflecting points through the origin and then rotates points clockwise through $\pi/2$ radians?

(c) Decide the dependence or independence of the vectors $\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$.
Explain your answer.

(d) Let **H** be the union of the first and third quadrants in the xy-plane.
That is, let $\mathbf{H} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$. Show that **H** is not a subspace of $\mathbf{R}^2$.

(e) Let T: $\mathbf{R}^2 \to \mathbf{R}^3$ be a transformation. Show that the transformation T defined by $T(x_1, x_2) = (5x_1, x_2 + 10, 6x_2)$ is not linear.

(f) Determine if $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in the column space of **A**, where $\mathbf{A} = \begin{bmatrix} 5 & 1 \\ -1 & 5 \end{bmatrix}$.
Explain your answer.