

科目：線性代數

系所組：電機系 丙組

1. Let  $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}$ . Find inverse of  $A$ . (10)

2. Find all solutions of the linear system, 
$$\begin{cases} x_1 - x_2 + 3x_3 + 2x_4 = 1 \\ -x_1 + x_2 - 2x_3 + x_4 = -2 \\ 2x_1 - 2x_2 + 7x_3 + 7x_4 = 1 \end{cases} \quad (10)$$

3. Compute the value of  $\det(A)$  if  $A = \begin{bmatrix} x & 1 & 1 \\ 1 & x & -1 \\ -1 & -1 & x \end{bmatrix}$ . (Your answer should be a function of  $x$ ).

For what values of  $x$  will the matrix be singular? (10)

4. Determine a basis for the null subspace of  $A$ , if  $A = \begin{bmatrix} 4 & -2 \\ 1 & 3 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$ . (10)

5. Find the least squares solution of the system : 
$$\begin{cases} -x_1 + x_2 = 10 \\ 2x_1 + x_2 = 5 \\ x_1 - 2x_2 = 20 \end{cases} \quad (10)$$

6. Find a basis for the subspace  $S$  of  $R^4$  consisting of all vectors of the form  $(a+b, a-b+2c, b, c)^T$ , where  $a, b$  and  $c$  are real numbers. (10)

7. Let  $S$  be the subspace of  $R^3$  spanned by  $\mathbf{x} = (1, -1, 1)^T$ . Find a basis for  $S^\perp$ . (10)

8. If  $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$ , find the eigenvalues and corresponding eigenspaces for  $A$ . (10)

9. Let  $S$  be the subspace of  $P_3$  consisting of all polynomials of the form  $ax^2 + bx + 2a + 3b$ . Find a basis for  $S$ . (10)

10. Let  $\mathbf{x} = \begin{bmatrix} 4 \\ 4 \\ -4 \\ 4 \end{bmatrix}$  and  $\mathbf{y} = \begin{bmatrix} 4 \\ 2 \\ 2 \\ 1 \end{bmatrix}$ . (a.) Determine the angle between  $\mathbf{x}$  and  $\mathbf{y}$ . (b.) Determine the distance between  $\mathbf{x}$  and  $\mathbf{y}$ . (10)

※ 注意：1.考生須在「彌封答案卷」上作答。

2.本試題紙空白部份可當稿紙使用。

3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。