

科目：通訊系統系所組：電機工程學系甲組

1. Consider an impulse train function  $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - 0.5n)$ .  $y(t)$  is a function of the impulse train as given by  $y(t) = -x(0.5t + 0.05) + 1$ . An ideal bandpass filter  $h(t)$  with frequency response in Hz is given by  $H(f) = \begin{cases} 1, & 4.5 < |f| < 5.5 \\ 0, & \text{otherwise} \end{cases}$ .
- (1) Find the exponential Fourier series of  $y(t)$ . (10%)
  - (2) Find the Fourier transform of  $y(t)$ . (10%)
  - (3) Pass  $y(t)$  through the ideal bandpass filter  $h(t)$ . The filter output is assumed to be  $z(t)$ . Find the Hilbert transform of  $z(t)$ . (10%)
2. A frequency modulation (FM) signal is expressed as  $x(t) = \cos(2000\pi t + 20\pi \sin(20\pi t))$ .
- (1) Find the peak frequency deviation in Hz. (5%)
  - (2) Determine the Carson's bandwidth in Hz. (5%)
  - (3) Find the average power of  $x(t)$ . (5%)
  - (4) Can we regard  $x(t)$  as a narrowband FM signal? Please explain your answer. (5%)
3. Let a random process be given as  $z(t) = x(t)y(t)$  where  $x(t)$  is a stationary random process with autocorrelation function  $R_x(\tau) = e^{-|\tau|}$  and  $y(t) = 10 \cos(200\pi t + \theta)$  with  $\theta$  uniformly distributed over the interval  $[0, 2\pi]$ .  $x(t)$  and  $y(t)$  are statistically independent.
- (1) Find the autocorrelation function of  $z(t)$ . (5%)
  - (2) Find the power spectral density of  $z(t)$ . (5%)
  - (3) Find the DC power in  $z(t)$ . (5%)
  - (4) Find the AC power in  $z(t)$ . (5%)
4. For a 2-level transmission system, the value of the transmitted data  $x$  is randomly picked from the set  $\{1, -1\}$ . The received data  $y$  is corrupted by an additive noise  $n$  with probability density function  $f(n) = \begin{cases} (n+a)/a^2, & -a \leq n < 0 \\ (a-n)/a^2, & 0 \leq n < a \\ 0, & \text{otherwise} \end{cases}$ . Thus, we can have the form of  $y = x + n$ . Assume a maximum a posteriori detector is employed. Find the probability of error. (10%)
5. For a 2-level transmission system, the value of the transmitted data  $x$  is either 1 or 4. The probability for  $x=1$  is  $p$  while for  $x=4$  is  $1-p$ . The data  $x$  is sent over a channel with an additive white Gaussian noise  $n$  with zero mean and unit variance. Then, the received data can be expressed as  $y = x + n$ . Assume a maximum a posteriori (MAP) detector is employed. When  $y < 2$ , the MAP detector output is 1. While  $y > 2$ , the MAP detector output is 4.
- (1) Determine  $p$ . (10%)
  - (2) Find the probability of error in terms of  $Q$  function. (10%)

[PS:  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du$ ]

※ 注意：1. 考生須在「彌封答案券」上作答。

2. 本試題紙空白部分可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。