科目： 近代物理　　系所組： 物理學系

1. (a) (20%) The potential energy for a particle in a one-dimensional box is defined as

$$V(x) = \begin{cases} 0 & \text{for } -a/2 < x < a/2 \\ \infty & \text{for } |x| > a/2 \, . \end{cases}$$

Find the normalized stationary-state wave functions $\Psi_n(x)$ and the energy eigenvalues E_n by solving the time-independent Schrödinger equation. Note that n is any integer, odd or even.

(b) (15%) Assume that the particle in the box is described by a superposition of the two lowest-energy eigenfunctions, i.e.,

$$\Psi(x) = \frac{1}{\sqrt{2}}(\Psi_1 + \Psi_2).$$

Calculate the expectation values $< x >$ and $< p >$, where p is the momentum operator.

2. Assume a square-well potential

$$V(x) = \begin{cases} 0 & \text{for } -a/2 < x < a/2 \\ V_0 & \text{for } |x| > a/2 \end{cases}$$

is capable of producing a few bound states.

(a) (15%) Sketch the qualitative behavior of the eigenfunctions for the ground state and the first two excited states.

(b) (20%) Derive the transcendental equation

$$\tan\sqrt{\frac{ma^2}{2\hbar^2}E} = \sqrt{\frac{V_0 - E}{E}}$$

for the energy eigenvalues of the *even* states in the finite square well.

3. (a) (15%) Sketch the *normal* Zeeman splittings for the $n = 2$ and $n = 3$ states of the hydrogen atom, and identify the allowed ($n = 3 \rightarrow n = 2$) electric dipole transitions.

(b) (15%) Sketch the *anomalous* Zeeman splittings for the $1S_{1/2}$, $2P_{1/2}$ and $2P_{3/2}$ energy levels of the hydrogen atom, and identify the allowed $2P_{1/2} \rightarrow 1S_{1/2}$ and $2P_{3/2} \rightarrow 1S_{1/2}$ electric dipole transitions.