科目：**線性代數** 系所組：**數學**

1. (10 points) Use Gaussian elimination to solve the system $\begin{cases} x-4y-z+w=3 \\ 2x-8y+z-4w=9 \\ -x+4y-2z+5w=-6 \end{cases}$.

2. Let $A=\begin{bmatrix} 3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. Compute

   (a) (10 points) the characteristic polynomial and the eigenvalues of A,

   (b) (15 points) a basis for each eigenspace of A, and

   (c) (5 points) the algebraic and geometric multiplicity of each eigenvalue.

3. Let $\mathsf{P}_n(\mathsf{R})$ consist of all polynomials over R having degree less than or equal to n. Define a function $\mathsf{T}:\mathsf{P}_2(\mathsf{R})\to\mathsf{P}_3(\mathsf{R})$ by $\mathsf{T}(f(x))=xf(x)+f'(x)$, where $f'(x)$ denotes the derivative of $f(x)$.

   (a) (10 points) Prove that T is a linear transformation.

   (b) (10 points) Compute the nullity and rank of T.

   (c) (10 points) Use the appropriate theorems to determine whether T is one-to-one or onto.

4. (10 points) Find an orthogonal basis for R^3 contains the vector $\mathbf{v}=(1,3,4)$.

5. (10 points) Let T be a linear operator on a finite-dimensional inner product space V. Prove that $\mathsf{N}(\mathsf{T}^*\mathsf{T})=\mathsf{N}(\mathsf{T})$, where $\mathsf{N}(\mathsf{T})$ is the null space of T and T^* is the adjoint operator of T.

6. (10 points) Let T be the linear operator on $\mathsf{M}_{2\times 2}(\mathsf{R})$ defined by $\mathsf{T}\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} c & d \\ a & b \end{pmatrix}$. Determine whether T is normal, self-adjoint, or neither.

※ 注意： 1. 考生須在「彌封答案卷」上作答。
2. 本試題紙空白部分可當稿紙使用，試題須隨答案卷繳回。
3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。