

東吳大學 102 學年度碩士班研究生招生考試試題

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系級	數學系碩士班	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (20 points) Consider the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 6 & 8 \end{pmatrix}$. Find a basis for the null space of A , and find a basis for the column space of A .

2. (20 points) Diagonalize the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix},$$

and compute A^k in terms of k .

3. (20 points) Apply Gram-Schmidt orthonormalization process to the three vectors $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$.

4. (20 points) Let $T : R^2 \rightarrow R^3$ be a linear transformation, and $T\left(\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$, $T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -6 \\ 3 \\ 2 \end{pmatrix}$. Find $T\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right)$.

5. (20 points) Find the least-squares fit function $y = c + dx$ to the points $(1, 1), (2, 6), (3, 11), (4, 12)$.