

東吳大學 102 學年度碩士班研究生招生考試試題

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系 級	數學系碩士班	考試 時間	100 分鐘
科 目	高等微積分	本科 總分	100 分

1. (15%) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\int_0^x \sin 3t \, dt}{x^2}.$$

2. (15%) Let a_n be a sequence of real numbers that converges to L . Show that a_n is a Cauchy sequence.

3. (20%) Let

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}.$$

- Show that f is continuous at $(0, 0)$.
 - Show that $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$ exist.
 - Is f differentiable at $(0, 0)$?
4. (30%) Evaluate the given integral by making an appropriate change of variables.
- $\iint_D \sin(x^2 + y^2) dx dy$ where $D = \{(x, y) | x^2 + y^2 \leq 9\}$.
 - $\iint_R \frac{x-2y}{3x-y} dx dy$ where R is the region bounded by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$, and $3x - y = 8$.

5. (20%) Given the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n,$$

- show that the radius of convergence is 1.
- show that the series converges uniformly on the interval $[-a, a]$, if $0 < a < 1$.
- find the sum of the series for $-1 < x < 1$.