

國立中央大學101學年度碩士班考試入學試題卷

所別：水文與海洋科學研究所碩士班 不分組(一般生) 科目：應用數學 共 4 頁 第 1 頁
 水文與海洋科學研究所碩士班 不分組(在職生)

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

(共 50 題，題目皆僅有單一正確選擇答案，每題 2 分)

1. What is the minimum period of $\sin 7x$? (A) 7π (B) $2\pi/7$ (C) 3.5π (D) $\pi/7$ (E) none of above
2. $\sin \frac{\pi}{6} =$ (A) $1/\sqrt{2}$ (B) $1/2$ (C) $\sqrt{3}/2$ (D) $1/3$ (E) $\pi/6$
3. The solutions of the equation $3x^3 + 4x^2 - 5x - 2 = 0$ are $x=1$, a and b , then $a+b =$
 (A) 2 (B) -2 (C) $-7/3$ (D) $7/3$ (E) none of above
4. $\frac{d(\cos 2x)}{dx} =$ (A) $\sin 2x$ (B) $2\sin 2x$ (C) $-\sin 2x$ (D) $-2\sin 2x$ (E) $-0.5\sin 2x$
5. $\frac{d^2 x^{2n}}{dx^2} =$ (A) $4n^2 x^{2n}$ (B) $4n^2 x^{2n-2}$ (C) $2n(2n-1)x^{2n-2}$ (D) $2n(n-1)x^{2n-2}$ (E) $2n(n-1)x^n$
6. The first derivative of $\ln t^2$ with respect to t is (A) e^{t^2} (B) $2t$ (C) $2t^2$ (D) $2t \ln t^2$ (E) $2/t$
7. If the first derivative of $(x-1)/(x+1)$ with respect to x is $(ax+b)/(x+1)^2$. What is the value of $(a-b)$?
 (A) -2 (B) 2 (C) 0 (D) 1 (E) 4
8. What is the integration of x^{2n} with respect to x , where c is a constant?
 (A) $x^{2n+1}/(2n+1)+c$ (B) $x^{2n+1}/(2n)+c$ (C) $x^{2n+1}+c$ (D) $2nx^{2n-1}+c$ (E) $x^{2n-1}+c$
9. $\int_0^{\pi/2} \sin 2x \, dx =$ (A) 0 (B) 1 (C) 2 (D) -1 (E) -2
10. The perimeter of a semi circle with a radius r is (A) $2\pi r$ (B) πr (C) $2\pi r + r$ (D) $\pi r + 2r$ (E) $2\pi r + 2r$
11. The area of a circle with a diameter d is (A) $\frac{\pi}{8} D^2$ (B) $\frac{\pi}{4} D^2$ (C) $\frac{\pi}{2} D^2$ (D) πD^2 (E) $2\pi D^2$
12. If the two straight lines $5x+4y=3$ and $ax+by=1$ are perpendicular to each other, then $(a-b)/(a+b) =$
 (A) -9 (B) 9 (C) -8 (D) 8 (E) 6
13. If m is the number of real solutions of the equation $e^x - x = 2$, then $m =$ (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
14. A real function is given by $f(x) = 3x^2 + ax + 2$, where a is a real number. If $f(1) = 6$, then $f(-1) =$
 (A) 1 (B) -2 (C) 20 (D) -13 (E) 4
15. $\int_2^6 |x-1| \, dx =$ (A) 5 (B) 4 (C) 3 (D) 2 (E) 1
16. What is the third term of the Taylor series of a real function $f(x)$ about $x = x_0$? (A) $\frac{1}{2} f''(x_0)(x-x_0)^2$
 (B) $\frac{1}{3} f''(x_0)(x-x_0)^2$ (C) $\frac{1}{3} f^{(3)}(x_0)(x-x_0)^3$ (D) $\frac{1}{6} f^{(3)}(x_0)(x-x_0)^3$ (E) none of above
17. Real function $f(x, y, z) = 2x^2 + yz^3$. The gradient $\nabla f =$ (A) $4x + z^3 + 3yz^2$ (B) $4x^2 + 3yz^3$
 (C) $4x\hat{i} + z^3\hat{j} + 3yz^2\hat{k}$ (D) $4x^2\hat{i} + 3yz^3\hat{k}$ (E) $4x\hat{i} + z^3\hat{j}$

注意：背面有試題

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*請在試卷答案卷(卡)內作答

18. Consider the differential equation $x(x+1)y'' + (2-x^2)y' = (2+x)y$ for $y = y(x)$. The differential equation is a
 (A) linear and homogeneous (B) nonlinear and homogeneous (C) linear and nonhomogeneous
 (D) nonlinear and nonhomogeneous (E) none of above

Consider the ordinary differential equation $\frac{dy}{dx} = 4y + x$, where $y = y(x)$ in questions 19 and 20.

19. The differential equation is (A) linear and homogeneous (B) nonlinear and homogeneous (C) linear and nonhomogeneous (D) nonlinear and nonhomogeneous (E) none of above

20. Which of the following is the solution of the differential equation?
 (A) $2e^{2x} - x + 1$ (B) $e^x + 2x + 6$ (C) $2e^{2x} + \frac{x}{2} + \frac{1}{4}$ (D) $e^{4x} - \frac{x}{4} - \frac{1}{16}$ (E) none of above

Consider the ordinary differential equation $x^2y' + 3xy = \frac{1}{x}$, where $y = y(x)$ in questions 21 and 22.

21. The differential equation is (A) linear and homogeneous (B) nonlinear and homogeneous (C) linear and nonhomogeneous (D) nonlinear and nonhomogeneous (E) none of above

22. The general solution is given as $y(x) = ax^{-2} + bx^{-3}$, where a and b are real constant. If $y(1) = -1$, then $a + b =$
 (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Consider the differential equation $\frac{d^2f}{dx^2} + \frac{df}{dx} - 2f = 0$ for $f = f(x)$ in questions 23 and 24.

23. Let a and b be real constants. The solution of the differential equation is in the form of (A) $f = ae^x + be^{2x}$
 (B) $f = ae^{-x} + be^{2x}$ (C) $f = ae^x + be^{-2x}$ (D) $f = ae^{-x} + be^{-2x}$ (E) $f = ae^x + be^{\sqrt{2}x}$.

24. If $f(0) = 4$ and $f'(0) = -5$ then $a + b =$ (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Consider the differential equation $y'' + 9y = \cos x$ for $y = y(x)$ in questions 25, 26, 27 and 28.

25. Then the general form of *homogeneous* solution is (A) $a + bx^3$ (B) $ae^{3x} + be^{-3x}$ (C) $a \cos 3x + b \sin 3x$
 (D) $e^{3x}(a \cos 3x + b \sin 3x)$ (E) none of above

26. The general form of the *particular* solution of the differential equation is (A) $A \cos x + B \sin x$
 (B) $(A + Bx) \cos x$ (C) $x(A \cos x + B \sin x)$ (D) $(A + Bx) \sin x$ (E) none of above

27. The two real constants A and B satisfy $A + B =$ (A) 1 (B) 0.5 (C) 2 (D) 0.25 (E) 0.125

28. If $y(0) = 0$ and $y'(0) = 3$, then $a \times b =$ (A) -1 (B) -0.5 (C) -2 (D) -0.25 (E) -0.125

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*請在試卷答案卷(卡)內作答

Consider the differential equation $y'' + 9y = \cos 3x$ for $y = y(x)$ in questions 29 and 30.

29. The general form of the *particular* solution of the differential equation is (A) $A \cos 3x + B \sin 3x$
 (B) $(A + Bx) \cos 3x$ (C) $x(A \cos 3x + B \sin 3x)$ (D) $(A + Bx) \sin 3x$ (E) none of above

30. The two real constants A and B satisfy $A + B =$ (A) 1 (B) $1/2$ (C) $1/3$ (D) $1/4$ (E) $1/6$

31. Which of the following is the equation of a line tangent to the curve of $y = 2x + \ln x$ at $x = 1$?

(A) $y = 2x + 1$ (B) $y = 3x - 1$ (C) $y = 0.5x + 2$ (D) $y = -x - 3$ (E) $y = x + 0.5$

32. f and g are twice-differentiable real functions. If $f'(x) > g'(x)$ for all $x > 0$, then

(A) $f(x) > g(x)$ (B) $f''(x) > g''(x)$ (C) $[f(x) - f(0)] > [g(x) - g(0)]$
 (D) $[f'(x) - f'(0)] > [g'(x) - g'(0)]$ (E) $[f''(x) - f''(0)] > [g''(x) - g''(0)]$, for all $x > 0$.

33. If the system of linear equations $\begin{cases} x + 4y = 0 \\ 2x - ay = 0 \end{cases}$ has *nontrivial* solution, then $a = ?$

(A) 8 (B) -8 (C) 12 (D) -12 (E) 9

34. Let $f(t) = e^{at}$ for $t \geq 0$, where a is a constant. The Laplace transform of $f(x)$ is

(A) $\frac{1}{s-a}$ (B) $\frac{1}{s+a}$ (C) $\frac{-1}{s+a}$ (D) $\frac{-1}{s-a}$ (E) none of above

35. The Laplace transform of $\sin \omega t$ is $\frac{\omega}{s^2 + \omega^2}$. What is the Laplace transform of $\cos \omega t$?

(A) $\frac{-\omega}{s^2 + \omega^2}$ (B) $\frac{\omega}{s^2 - \omega^2}$ (C) $\frac{s}{s^2 + \omega^2}$ (D) $\frac{-s}{s^2 + \omega^2}$ (E) none of above

36. If matrices $[A] = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $[B] = \begin{bmatrix} 4 & 0 \\ 1 & 3 \end{bmatrix}$, $[A] \times [B] = ?$

(A) $\begin{bmatrix} 2 & -2 \\ 2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & -3 \\ 3 & 9 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & -3 \\ 9 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} 9 & -2 \\ 2 & 3 \end{bmatrix}$ (E) $\begin{bmatrix} 2 & -2 \\ -5 & 4 \end{bmatrix}$

37. The directional derivative of $f(x, y, z) = 2x + y^2 + 3z^3$ at the point $(1, 2, -1)$ in the direction of $\vec{e} = \hat{i} + \hat{j} - \hat{k}$ is

(A) 3 (B) $\sqrt{3}$ (C) $-\sqrt{3}$ (D) $2\sqrt{3}$ (E) $-2\sqrt{3}$

38. What is the unit normal vector \hat{n} of the surface $z^2 = x^2 + y^2$ at the point $(1, -1, 1)$? (A) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

(B) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$ (C) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (D) $\frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} + \hat{k})$ (E) $\frac{1}{\sqrt{3}}(-\hat{i} - \hat{j} - \hat{k})$

39. What is the divergence of a vector $3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$ at the point $(1, 1, 1)$? (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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Let the matrix $[A] = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ in questions 40 and 41.

40. The determinant of $[A]$ is (A) 21 (B) -8 (C) 6 (D) 9 (E) 15

41. The inverse of $[A]$ is (A) $-\frac{1}{15} \begin{bmatrix} 7 & 1 & -3 \\ 4 & 0 & 1 \\ 1 & -14 & 2 \end{bmatrix}$ (B) $\frac{1}{21} \begin{bmatrix} 5 & 1 & 7 \\ -7 & 7 & -14 \\ -6 & 3 & 0 \end{bmatrix}$ (C) $-\frac{1}{8} \begin{bmatrix} 0 & -7 & -6 \\ 1 & 7 & 3 \\ 7 & -14 & 0 \end{bmatrix}$

(D) $\frac{1}{6} \begin{bmatrix} 0 & -7 & -6 \\ 1 & 3 & 3 \\ 2 & -14 & 0 \end{bmatrix}$ (E) $\frac{1}{9} \begin{bmatrix} 5 & -7 & -6 \\ 1 & 2 & 3 \\ 6 & -14 & 5 \end{bmatrix}$

42. What is the curl of a vector $yz\hat{i} + 2xz\hat{j} + z\hat{k}$? (A) $2y\hat{i} + 2x\hat{j} + x\hat{k}$ (B) $x\hat{i} - 2y\hat{j} + z\hat{k}$ (C) $z\hat{i} - 2x\hat{j} - y\hat{k}$
 (D) $x\hat{i} - y\hat{j} + 2z\hat{k}$ (E) $-2x\hat{i} + y\hat{j} + z\hat{k}$

43. Let λ_1 and λ_2 be eigenvalues of the matrix $[A] = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$. $\lambda_1 + \lambda_2 =$ (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

44. Which of the following is one of the eigenvector of the matrix $[A]$ in question 43?

(A) $\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$ (B) $\begin{Bmatrix} -1 \\ 2 \end{Bmatrix}$ (C) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ (D) $\begin{Bmatrix} 1 \\ 3 \end{Bmatrix}$ (E) $\begin{Bmatrix} 3 \\ 1 \end{Bmatrix}$

45. Which of the following is another eigenvector of the matrix $[A]$ in question 43?

(A) $\begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$ (B) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (C) $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ (D) $\begin{Bmatrix} 1 \\ -3 \end{Bmatrix}$ (E) $\begin{Bmatrix} 3 \\ -1 \end{Bmatrix}$

46. A tetrahedron is determined by three edges vector $\vec{a} = [2, 0, 3]$, $\vec{b} = [0, 2, 1]$, $\vec{c} = [4, 5, 0]$. The volume of the tetrahedron is (A) 5/3 (B) 11/3 (C) 5 (D) 17/3 (E) 19/3

47. If $w = x^2 - y^2$, $x = u + 2v$, $y = 2u - v$. What is $\partial w / \partial u$ at $x=1, y=1$? (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

48. As in question 47, what is $\partial w / \partial v$ at $u=1, v=-1$? (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

49. $\int_0^1 \int_0^{2x} (x+2y) dy dx =$ (A) 1/3 (B) 2/3 (C) 1 (D) 4/3 (E) 5/3

50. In the xy -plane, the curve with parametric equations $x = 2\cos t$ and $y = 2\sin t$, $0 \leq t \leq \pi/3$, has length (A) 2/3 (B) $2\pi/3$ (C) 1/3 (D) $\pi/3$ (E) $\sqrt{6}$

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