國立臺南大學 102 學年度 **綠色能源科技學系碩士班** 招生考試 工程數學 試題卷

1. Solve for x. $\begin{vmatrix} x & -1 \\ 3 & 1-x \end{vmatrix} = \begin{vmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{vmatrix}$ (10 分)

2. Solve the differential equation: $[x + y\cos(x)]dx + \sin(x)dy = 0$ (10 分)

3. Find the value of the integral $\int_0^{\infty} t^2 \sin(2t) e^{-3t} dt$ (10 分)

4. Evaluate the value of the integral $\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta}$ (10 分)

5. Let $\vec{v_1} = (1,\ 2,\ 1)$, $\vec{v_2} = (2,\ 9,\ 0)$, and $\vec{v_3} = (3,\ 3,\ 4)$. Show that the set $S = (\vec{v_1},\ \vec{v_2},\ \vec{v_3})$ is a basis for $\boldsymbol{R^3}$. (10 分)

6. Find the inverse Laplace transform of $F(s) = \frac{1}{(s^2+1)(s^2+9)}$ (10 分)

7. Let $A = \begin{bmatrix} 4 & -8 & -6 \\ 0 & 1 & 0 \\ 1 & 8 & 9 \end{bmatrix}$, find the eigenvalues and eigenvectors. (10 分)

8. Solve $u(x,t)$ of following partial differential equation (20 分)

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < \pi,\ t > 0$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0$$
$$u(x,0) = 2 + \cos x - 3\cos 3x$$

9. Find the general solution of the ordinary differential equation (10 分)

$$\frac{dy}{dx} + x^2 y = x^2 y^3$$