

每大題 10 分，共 10 題，合計 100 分

1. Let the matrix  $A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$ . Find the inverse of the matrix  $A$ .

(if it exists)

2. Evaluate  $y^{-3}y' + y^{-2} = 1$ .

3. Compute the inverse Laplace transform of the function  $\frac{e^{-s} + e^{-2s}}{s^2 - 3s + 2}$ .

4. Suppose the function  $f$  has fundamental period  $p=4$ , and

$$f(x) = \begin{cases} x^2 & , 0 \leq x < 1 \\ 1 & , 1 \leq x \leq 4 \end{cases}$$

Write the Fourier series for the function  $f$ .

5. Evaluate  $\oint_{|z|=2} \frac{\sin z}{z^2 + 1} dz$  with Cauchy Integral Formula.

6. The signal  $x(t) = \sin(2\pi t) + \cos(3\pi t)$  is represented by the Fourier

series as  $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$ . Evaluate the Fourier series coefficients  $C_n$

of the signal  $x(t)$ .

7. Consider the basis  $S = \{\vec{v}_1, \vec{v}_2\}$  for  $\mathbb{R}^2$ , where  $\vec{v}_1 = (1, 1)$  and  $\vec{v}_2 = (1, 0)$  and let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear operator such that  $T(\vec{v}_1) = (1, -2)$  and  $T(\vec{v}_2) = (-4, 1)$ .

(a) Find a formula for  $T(x_1, x_2)$  (5 points)

(b) Find  $T(5, -3)$  (5 points)

8. Solve the differential equation:

$$x \frac{d^2 y}{dx^2} - (2x + 1) \frac{dy}{dx} + (x + 1)y = (x^2 + x - 1)e^{2x}$$

9. Evaluate the value of the integral  $\int_0^\pi \frac{d\theta}{(3 + 2\cos\theta)^2}$ .

10.(a) Find a matrix  $\mathbf{P}$  that diagonalizes  $\mathbf{A} = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ . (5 points)

(b) Let the matrix  $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ . Find  $e^{\mathbf{A}}$ . (5 points)