© 每大題 $25 \%$ ，合計 4 大題，共計 $100 \%$ 。
1．Fig． 1 shows two blocks of a control system and the differential equations relating input and output．Assumed all initial conditions for $b$ and $y$ are zero，find the Lapalce transform of the transfer function between $x(t)$


Fig． 1

2．Consider a system defined by the following state－space equations：

$$
\left[\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2}
\end{array}\right]=\left[\begin{array}{cc}
-5 & -1 \\
3 & -1
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]+\left[\begin{array}{l}
2 \\
5
\end{array}\right] u, \quad y=\left[\begin{array}{ll}
1 & 2
\end{array}\right]\left[\begin{array}{l}
x_{1} \\
x_{2}
\end{array}\right]
$$

obtain the transfer function $\mathrm{G}(\mathrm{s})$ of the system．

3．Give the quadratic function as follows．
$V(X)=10 x_{1}^{2}+4 x_{2}^{2}+x_{3}^{2}+2 x_{1} x_{2}-2 x_{2} x_{3}-4 x_{1} x_{3}$, where $X$ is the state vector．
（a）Put the $V(\mathrm{X})$ in the matrix form of $V(X)=X^{T} P X$ ．What is the matrix $P$ ？
（b）Use Sylvester＇s criterion to verify whether or not the $V(X)$ is positive definite．

4．Please sketch the root－locus plot of a system with complex－conjugate open－loop poles．

$G(s)=\frac{K(s+2)}{\left(s^{2}+2 s+3\right)}, H(s)=1$ ，The poles of $\mathrm{G}(\mathrm{s})$ are $s=-1 \pm j \sqrt{2}$

