

1. Find a general solution for

$$x^2 y' = y^2 + 2xy$$

(25%)

2. Use the matrix exponential to solve the initial value problem $\dot{Y} = AY$, $Y(0) = Y_0$ where

$$A = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix} \text{ and } Y_0 = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

(25%)

3. Find the Laplace transform of the function (25%)

$$f(t) = \begin{cases} 1 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \cos t & \text{if } t > 2\pi \end{cases}$$

4. A is a 3x3 real symmetric matrix. Try to find out its eigenvalues and corresponding eigenvectors, respectively. (25%)

$$A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$