國立中央大學101學年度碩士班考試入學試題卷

所別：大氣科學學系大氣物理碩士班 不分組(一般生)　科目：應用數學　共 1 頁 第 1 頁
大氣科學學系大氣物理碩士班 不分組(在職生)

本科考試禁用計算器　　＊請在試卷答案卷（卡）內作答

1. Explain and compare the following terms.
   a. Homogeneous and non-homogeneous differential equations;
   b. Gradient, Divergence, and Curl;
   c. Symmetric, Skew-symmetric , and Orthogonal matrices;
   d. Initial value problem and boundary value problem;
   e. Power series, Taylor series, and Laurent series.

   (15%)

2. Find general solutions of the following equations

   a. $2xyy' + (x-1)y^2 = x^2 e^x$,

   b. $y'' - 4y' + 5y = e^{2x}\csc x$,

   c. $(x^2D^2 - 4xD + 6I)y = 21x^{-4}$

   (15%)

3. Determine the type and stability of the critical point. Then find a real general solution.

$$y_1' = 3y_1 + 5y_2$$
$$y_2' = -5y_1 - 3y_2$$

(10%)

4. Solve the following initial value problem by Laplace transform

$$y'' + y' - 2y = \begin{cases} 3\sin t - \cos t & 0<t<2\pi \\ 3\sin 2t - \cos 2t & t>2\pi \end{cases}, \quad y(0) = 0, \quad y'(0) = -1$$

(10%)

5. Find the eigenvalues and eigenvectors of the following matrices

$$\begin{bmatrix} -3 & 0 & 4 & 2 \\ 0 & 1 & -2 & 4 \\ 2 & 4 & -1 & -2 \\ 0 & 2 & -2 & 3 \end{bmatrix}$$

(10%)

6. Let $f = xy - yz$, $\mathbf{v} = [4z \quad 2y \quad x-z]$, $w = [y^2 \quad y^2 - x^2 \quad 2z^2]$. Find

   a. $f\nabla f$ at $P:(0,\ 3,\ 1)$　b. $\nabla^2(xzf)$　c. $\nabla\cdot(\mathbf{v}\times\mathbf{w})$

   d. $\nabla(\nabla\cdot\mathbf{w})$　e. $\nabla\times\mathbf{w}\cdot\mathbf{v}$ at $(4,\ 0,\ 2)$

   (15%)

7. Represent $f(x,y)$ by a double Fourier series.
   $f(x,y) = xy(a-x)(b-y)$　$(0<x<a, 0<y<b)$

   (10%)

8. What is the d'Alembert's method? To what equation does it apply? What are elliptic, parabolic, and hyperbolic equations? Give an example for each type of equation.

   (15%)