

一、Let  $V$  be  $R^3$ , i.e., three dimensional space.

(1) Show that  $W = \{(a, b, c) \mid 2a + 3b + 4c = 0, a, b, c \in R\}$  is a subspace of  $V$ . (5%)

(2) Find the basis of  $W$ . (5%)

二、Let  $T : R^4 \rightarrow R^3$  be a function defined by

$$T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t).$$

(1) Show that  $T$  is a linear transformation. (5%)

(2) Find the image of  $T$ . (5%)

(3) Find the kernel of  $T$ . (5%)

三、Let  $T : R^2 \rightarrow R^2$  be given by  $T(x, y) = (2x, 2x + 3y)$ .

(1) Find the matrix representation of  $T$  with respect to the basis

$$B = \{(1, 1), (0, 2)\} \text{ for } R^2. (5\%)$$

(2) Then, using this matrix, find the matrix representation of  $T$  with respect to the basis  $B' = \{(1, 2), (0, 1)\}$ . (5%)

四、(1) 說明何為一個 matrix 的 rank(秩)。 (5%)

(2) 一個  $n$  階方陣什麼時候 rank 為  $n$ ? (5%)

(3) 求出下列方陣的 rank:  $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ . (5%)

五、 $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ , 求  $A$  之

(1) 特徵多項式(characteristic polynomial) (8%)

(2) 特徵值(eigenvalues) (6%)

(3) 特徵向量(eigenvectors) (6%)

六、(1) What is a nonabelian group? (5%)

(2) Give an example of the nonabelian group and prove it. (10%)

七、(1) 何謂一個 ring 的特徵(characteristic)? (5%)

(2) 找出下列各個 ring 的 characteristic:  $2Z$ 、 $Z \times Z$ 、 $Z_3 \times Z_3$ 、 $Z_3 \times 3Z$ 、 $Z_6 \times Z_{15}$ . (10%)