1. (10%)Let $f(x) = \begin{cases} 3x-5 & \text{if } x \leq -1 \\ -x+2 & \text{if } -1 < x \leq 2 \\ -3x^2 & \text{if } 2 < x \end{cases}$, and

   $g(x) = \begin{cases} -x+1 & \text{if } x < -1 \\ -x^2+3 & \text{if } -1 \leq x \leq 2 \\ 3-2x & \text{if } 2 < x \end{cases}$.

   Find $\lim_{x \to -1^-} f(g(x))$ and $\lim_{x \to -1^+} f(g(x))$.

2. (10%)Prove that $\lim_{n \to \infty} \cos\sqrt{1+4n^2\pi^2} = 1$.

3. (10%)Find the volume of the largest right circular cylinder (正圓柱體) that can be inscribed is a sphere of radius 3.

4. (10%)Find $\frac{d}{dx} \int_{t^2}^{2t} \sin\sqrt{t}\, dt$.

5. (10%)Find the volume of the solid generated by revolving the region
   $$\Omega = \{(x,y)|\ |x+y-10| + |2x-y-5| \leq 3\}$$
   about the x-axis.

6. (10%)Find $\sum_{n=0}^{\infty} \frac{n^2}{3^n}$

7. (10%)Let $a, b, c \in \mathbb{N}$. Find three positive numbers x, y and z whose sum is $a+b+c$ such that $x^a y^b z^c$ is a maximum.

8. (20%) Prove or disprove the following statements.
   (a) If $f : [0,1] \to R$ is a increasing function, then f is Riemann integrable on $[0,1]$.
   (b) Let X is a metric space and $E \subseteq X$. If E is closed and bounded, then E is compact.

9. (10%) Let
   $$f(x) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^2}.$$
   Prove that
   $$\int_0^{\pi/2} f(x)\, dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3}.$$