多選題 (每題 5 分，答錯每選項倒扣 1 分)

1. Suppose x, y represents people. Let $L(x,y)$ be the predicate of "x likes y," $F(x,y)$ be the predicate of "x is a friend of y," $C(y)$ be the predicate of "y is a capable person." Choose the correct logic statement(s) which have the same meaning as the sentence --- **No capable person is liked by all his friends**.
   (a) $\forall x\ (\forall y,\ (F(y,x) \rightarrow L(y,x)) \rightarrow \neg\, C(x))$.
   (b) $\exists x\ (\forall y,\ (L(x,y) \wedge F(y,x)) \rightarrow \neg C(x))$.
   (c) $\forall x\ (\forall y,\ (L(y,x) \rightarrow (\neg F(x,y))\ ) \rightarrow C(x))$.
   (d) $\neg\exists x,\ (C(x) \rightarrow (\forall y,\ F(y,x) \rightarrow L(y,x))$.
   (e) none of the above.

2. Among the following options, which are sufficient but not necessary conditions for the corresponding goals?
   (a) In a connected simple undirected graph, "a connected graph without any circuit", for "the connected graph is a tree".
   (b) For comparing the order of growth of two function f and g, "f is $o(g)$" for "f is $O(g)$ but not $\Theta(g)$".
   (c) For comparing two infinite set A and B, " there exists an 1-to-1 mapping from A to B", for "$|A| = |B|$".
   (d) "A group of 73 students to choose 10 classes", for "at least one class has more than or equal to 8 students".
   (e) For a finite set A with 20 elements, "considering $F = \{X \mid X$ is a subset of $A\}$, and set relation $\subseteq$", for "all the elements from F and the chosen relation forms a lattice".

3. To analyze the complexity of the following procedure P, We will use the following assumptions: Suppose P and Q are both procedures. Q takes $\theta(m)$ time to compute, where m is the size of input; each statement line in and outside the loop counts 1 step.

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Procedure P( array1[ a_1, a_2, ..., a_n ])
if n<4 exit.
declare initially new empty array2, array3;
for (i=1 to n)
{   if ( ((i mod 4) =2) or ((i mod 4) =0) )
        { insert  a_i  into array2;}
    else { insert  a_i  into array3;}
    if ( i ≤ √n )
        call Q(array1[ a_1, a_2, ..., a_i ]);   }
call P( array2 );
call P( array3 );
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Suppose n is a multiple of 4, which of the following relations on C_n can describe the time complexity of procedure P with respect to problem size n?

(a) $C_n = 2C_{n/2} + \theta(n)$

(b) $C_n = 4C_{n/4} + \theta(n)$

(c) $C_n = 2C_{n/4} + \theta(n^2)$

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(d) $C_n = 2C_{n/4} + \theta(n^2) + \theta(1)$

(e) none of the above

4. What can be the time complexity level of the procedure P in the question above?

(a) $O(n)$ (b) $O(n \log n)$ (c) $O(n^2)$ (d) $O(n^{\log_4^2})$ (e) none of the above

5. To solve the recurrence relation $a_{n+2} = a_{n+1} + 2a_n + (-1)^n$, $n \geq 0, a_0 = 1, a_1 = 1$, what of the followings will be the corresponding generating function $f(z)$?

(a) $f(z) = (1 + z + z^2)/(1 - 4z + 5z^2 - 2z^3)$.

(b) $f(z) = (1 - z + z^2)/(1 - 3z^2 + 2z^3)$.

(c) $f(z) = \frac{2}{1-2z} + \frac{-z}{(1+z)^2}. + \frac{2}{(1+z)^2}$

(d) $f(z) = \frac{2}{1-z} + \frac{2-3z}{(1+2z)^2}$.

(e) none of the above.

6. Suppose u_n and v_n are sequences defined recursively by $u_1 = 0$, $v_1 = 1$, and for $n \geq 1$, $u_{n+1} = \frac{1}{2}(u_n + v_n)$, $v_{n+1} = \frac{1}{4}(u_n + 3v_n)$.

(a) $v_n - u_n = \frac{1}{4^{n-1}}$ for $n \geq 1$.

(b) u_n is a decreasing sequence; that is, $u_{n+1} < u_n$ for all $n \geq 1$.

(c) v_n is a decreasing sequence; that is $v_{n+1} < v_n$ for all $n \geq 1$.

(d) $u_n = \frac{2}{3} - \frac{1}{6}(\frac{1}{4^{n-2}})$ for all $n \geq 1$.

(e) none of the above.

7. Which of the following statements are true?

(a) $f(n,m) = 2n + 3m$; $f: N \times N \to N; f$ is not onto, but it is one-to-one.

(b) $f(n,m) = 14n + 22m$; $f: N \times N \to N; f$ is neither onto nor one-to-one.

(c) $f(n,m) = 89n + 246m$; $f: Z \times Z \to Z$; f is not one-to-one, but it is onto.

(d) $f(n,m) = n^2 + m^2 + 1$; $f: Z \times Z \to N$; f is neither onto nor one-to-one.

(e) $f(n,m) = \left\lfloor \frac{n}{m} \right\rfloor + 1$; $f: N \times N \to N$; is not onto, but it is one-to-one.

8. Let $v_1, v_2, \ldots, v_8$ and $w_1, w_2, \ldots, w_{12}$ be the bipartition sets of the complete bipartite graph $K_{8,12}$.Let A be the adjacency matrix of this graph, where the vertices are listed in the order $v_1, v_2, \ldots, v_8, w_1, w_2, \ldots, w_{12}$.

(a) (1,5) entry of A is 1.

(b) (8,9) entry of A is 1.

(c) (10,12) entry of A^2 is 8.

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(d) (5,5) entry of A^2 is 0.
(e) (20,6) entry of A^2 is 0.

9. You and a friend meet three other couples at a party and several handshakes take place. Nobody shakes hands with himself or herself, there are no handshakes within couples, and no one shakes hands with the same person more than once. The numbers of hands shaken by the other seven people (excluding you) are all different.
(a) you shook three hands.
(b) your friend shook four hands.
(c) you shook four hands
(d) your friend shook three hands.
(e) you shook five hands.

10. Which of the following statements are true?
(a) If a and b are integers with $a-b\geq 0$ and $b-a\geq 0$, then $a=b$.
(b) If a and b are integers with $a-b>0$ and $b-a<0$, then $a=b$.
(c) $q\to(p\to q)$ is a tautology.
(d) $[(p\to q)\wedge(q\to r)]\to(p\to r)$ is a contradiction.
(e) None of the above.

11. Determine which sets of functions are linearly dependent.
(a) $f_1(t)=1$, $f_2(t)=\sin t$, and $f_3(t)=\cos t$.
(b) $f_1(t)=1$, $f_2(t)=\sin^2 t$, and $f_3(t)=\cos^2 t$.
(c) $f_1(t)=\cos 2t$, $f_2(t)=\sin^2 t$, and $f_3(t)=\cos^2 t$.
(d) $f_1(t)=t+t^2+t^3$, $f_2(t)=t^3+t^4+t^5$, and $f_3(t)=t^5+t^6+t^7$.
(e) none of the above.

12. Let X and Y be two sets of vectors in vector space V. Which of the following statements are true?
(a) If each $\mathbf{x}$ in X is in Span(Y), and each $\mathbf{y}$ in Y is in Span(X), then X and Y have the same span.
(b) Let V^X denote the set of all mappings from X to V. Then V^X is a vector space.
(c) If $X\subseteq Y$, then Span(X) $\subseteq$ Span(Y).
(d) If $X\subseteq Y$ and Y spans V, then so does X.
(e) none of the above.

13. Let $\mathbf{A}$ be an $m\times n$ matrix and $m<n$. Which of the following statements are true?
(a) If $\mathbf{Ax}=\mathbf{b}$ is a consistent system of equations, then the system have many solutions.
(b) If kernel of $\mathbf{A}$ is $\mathbf{0}$, then rank($\mathbf{A}$)=m.
(c) If $\mathbf{Ax}=\mathbf{0}$ for some nonzero vector $\mathbf{x}$, then the rank of $\mathbf{A}$ is less than n.
(d) If $\mathbf{u}$ and $\mathbf{v}$ are two distinct solutions for $\mathbf{Ax}=\mathbf{b}$, then for all real values of t, $t\mathbf{v}+(1-t)\mathbf{u}$ is also a solution.
(e) none of the above.

14. Let $\mathbf{A}$ be an $n\times n$ matrix. Which of the following statements are true?
(a) det($\mathbf{A}$)>0 if and only if $\mathbf{A}$ is invertible.
(b) The row vectors of $\mathbf{A}$ are linearly independent.
(c) det($\mathbf{EA}$)=det($\mathbf{E}$)det($\mathbf{A}$), where $\mathbf{E}$ is an $n\times n$ elementary matrix.
(d) $(\mathbf{A}^T)^{-1}=(\mathbf{A}^{-1})^T$ if $\mathbf{A}$ is invertible.
(e) none of the above.

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15. Let $\mathbf{A}=\begin{bmatrix}1 & 1 & -4 & -3\\ 2 & 0 & 2 & -2\\ 2 & -1 & 3 & 2\end{bmatrix}$. Which of the following statements are true?

(a) {(1,1, −4, −3), (0,1, −5, −2), (0, 0, 1, −1/2)} is a basis for the row space of **A**.
(b) {(1,2, 2), (1, 0, −1), (−4, 2, 3)} is a basis for the column space of **A**.
(c) {(1,9,1,3)} is a basis for the nullspace of **A**.
(d) {(1,1, −4, −3), (2,0, 2, −2), (2, −1, 3, 2)} is a basis for the row space of **A**.
(e) none of the above.

16. A is an $n\times n$ square matrix. λ_i's and e_i's are eigenvalues and eigenvectors of A, respectively.
(a) A always has n eigenvalues.
(b) A may have zero eigenvalue.
(c) A may have zero eigenvector.
(d) all λ_i's $\neq 0$, then A is invertible.
(e) all e_i's are linearly independent, then all λ_i's are distinct.

17. For an $n\times n$ square matrix A. λ_i's are eigenvalues of A.
(a) λ_i's may be complex number.
(b) $A = A^T$, then λ_i's are real.
(c) $A = A^T$, then λ_i's are positive complex.
(d) $A = A^T$, then λ_i's are positive real.
(e) all eigenvalues of A^TA are positive real.

18. If χ is the least-squares solution of the linear system $Ax = b$.
(a) $\chi = (A^TA)^{-1}A^Tb$.
(b) (A^TA) is always invertible.
(c) χ is always exists.
(d) χ is unique.
(e) $\chi = R^{-1}Q^Tb$ for any A.

19. Give the necessary conditions for a matrix A that can be decomposed into
$A = \lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T + .. + \lambda_n u_n u_n^T$,
where λ_i and u_i are eigenvalues and eigenvectors of A.
(a) A is a square matrix.
(b) A is a symmetric matrix.
(c) A can be decomposed into B^TB.
(d) A has linearly independent columns.
(e) A is orthogonally diagonalizable.

20. Find a singular value decomposition $A = U\Sigma V^T$ with U and V being both orthogonal matrices, where $A=\begin{bmatrix}4 & -2\\ 2 & -1\\ 0 & 0\end{bmatrix}$. Which values are **not** in U or V matrices ?
(a) 1.
(b) $1/\sqrt{3}$.
(c) $-1/\sqrt{3}$.

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