國立高雄大學 102 學年度研究所碩士班招生考試試題

系所：
科目：工程數學　　土木與環境工程學系(土木工程組)　　是否使用計算機：是
考試時間：100 分鐘
本科原始成績：100 分

1. (20%)

A temperature distribution function $T(x,y,z)$ in the space is inversely proportional to the distance to the origin. If $T(0,0,1)=50$，please determine (a) the temperature variation rate on the point $(2,3,3)$ in the direction of $3\hat{i}+\hat{j}+\hat{k}$ (b) the direction of maximum temperature growth rate on the point of $(2,3,3)$ and the growth rate?

2. (10%)

Please determine the length from $t=0$ to $t=2\pi$ of the following curve:

$$x=\cos t,\ \ y=\sin t \ \text{ and } \ z=kt$$

3. (20%)

$A=\begin{bmatrix} 5-\lambda & 4 & -2 \\ 4 & 5-\lambda & -2 \\ -2 & -2 & 3-2\lambda \end{bmatrix}$，please show the relationship between Rank (A) and λ?

4. (30%)

Consider an differential equation $mx''(t)+kx(t)=f(t)$ with an initial conditions $x(0)$ and $x'(0)$.

Show $x(t)=x(0)\cos(\omega t)+\dfrac{x'(0)}{\omega}\sin(\omega t)+\dfrac{1}{m\omega}\int_0^t \sin(\omega\tau)\,f(t-\tau)\,d\tau$, where $\omega=\sqrt{k/m}$.

5. (20%)

Solve the equation $y''+y=0$ by the method of power series. Then show $y(x)=a_0y_1(x)+a_1y_2(x)$,

$y_1(x)=1-\dfrac{1}{2!}x^2+\dfrac{1}{4!}x^4-...$, and $y_2(x)=x-\dfrac{1}{3!}x^3+\dfrac{1}{5!}x^5-...$