國立高雄大學102學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100分鐘

系所：
電機工程學系(通訊專業領域)　是否使用計算機：是
本科原始成績：100分

● 共十題，每題十分。請依題號順序作答，否則酌予扣分。

1. Consider the vector space R^n over R with the usual operations. Let

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ \cdots,\ e_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

and

$$\varepsilon_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ \varepsilon_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ \cdots,\ \varepsilon_n = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

form two bases of R^n.

(a) Find a matrix A such that $A[\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n] = [e_1, e_2, \cdots, e_n]$.

(b) If $v \in R^n$ has the coordinate $(1, 2, \cdots, n)$ on the basis $\{e_1, e_2, \cdots, e_n\}$, what is the coordinate of v under $\{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n\}$?

2. Construct a 3×3 real symmetric matrix A such that the eigenvalues of A are 1, 1, and -1, and $\alpha = [1,1,1]^t$ and $\beta = [2,2,1]^t$ are eigenvectors corresponding to the eigenvalue 1.

3. Define mapping T and U on the vector space R^n by

$$T(x_1, x_2, \cdots, x_n) = (0, x_1, x_2, \cdots, x_{n-1})$$

and

$$U(x_1, x_2, \cdots, x_n) = (x_n, x_1, x_2, \cdots, x_{n-1}).$$

(a) Find $UT(x_1, x_2, \cdots, x_n)$.

(b) Find $T^n(x_1, x_2, \cdots, x_n)$.

(c) Find matrix representation of T.

(d) Find matrix representation of U.

(e) Find dimension of $Ker(T)$.

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4. Let

$$V=\left\{x=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\in R^4 \mid x_1=x_3+x_4, x_2=x_3-x_4\right\}.$$

(a) Show that V is a subspace of R^4.

(b) Find a basis for V and for $V^{\perp}$.

5. For $P_3[x]$ over R, define the inner product as

$$\langle f,g\rangle=\int_{-1}^{1} f(x)g(x)dx$$

where $P_3[x]=\text{Span}\{1,x,x^2,x^3\}$.

(a) Find an orthonormal basis for the subspace $\text{Span}\{x,x^2\}$.

(b) Extend the basis in (a) to an orthonormal basis for $P_3[x]$ with respect to the inner product.

6. If X and Y are independent random variables both uniformly distributed on $(0,1)$, then calculate the probability density of $X+Y$.

7. If X and Y are independent Poisson random variables with respective means λ_1 and λ_2, calculate the conditional expected value of X given that $X+Y=n$.

8. Suppose the joint density of X and Y is given by

$$f(x,y)=\begin{cases}4y(x-y)e^{-(x+y)}, & 0<x<\infty,\ 0\le y\le x\\ 0, & \text{otherwise}\end{cases}$$

Compute $E[X|Y=y]$.

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9. Consider a circle of radius R, and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the point. (In other words, the point is uniformly distributed within the circle.) If we let the center of the circle denote the origin and define X and Y to be the coordinates of the point chosen, then, since (X,Y) is equally likely to be near each point in the circle, it follows that the joint density function of X and Y is given by

$$f(x,y)=\begin{cases} c, & \text{if } x^2+y^2\le R^2 \\ 0, & \text{if } x^2+y^2>R^2 \end{cases}$$

for some value of c.

(a) Determine c.

(b) Find the marginal density function of X.

(c) Find the marginal density function of Y.

(d) Compute the probability that D, the distance from the origin of the point selected, is less than or equal to a.

(e) Find $E[D]$.

10. Suppose you arrive at a post office having two clerks at a moment when both are busy but there is no one else waiting in line. You will enter service when either clerk becomes free. If service times for clerk i are exponential with rate λ_i, $i=1,2$, find $E[T]$, where T is the amount of time that you spend in the post office.