科目：工程數學　　　　　系所：應用物理學系
考試時間：100 分鐘　　　本科原始成績：100 分　　　是否使用計算機：是

1. Solve the differential equation $(x^4 + y^2)dx - xy\,dy = 0$ (10%)

2. Solve the differential equation $y'' - 10y' + 25y = 30x + 3$. (10%)

3. Find an equation of the plane that contains $(1, 0, -2)$, $(3, 1, 4)$, and $(2, -1, 0)$. (10%)

4. Use the Laplace transform to solve the initial-value problem. (10%)
$$y'' - 2y' - 3y = 0, \quad y(1) = -3, \quad y'(1) = -17$$

5. Find the eigenvalues and eigenvectors of the given matrix (10%)
$$\mathbf{A} = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 5 \end{bmatrix}.$$

6. A matrix is given by $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$.

    (a) Find the inverse $\mathbf{A}^{-1}$. (10%)
    (b) Find the determinant $\det \mathbf{A}^{-1}$. (5%)

7. Find the Fourier series of the function $f(x) = 3 - 2x$, where $-\pi < x < \pi$. (10%)

8. Evaluate the integral $\int_0^{2\pi} \dfrac{\sin^2 \theta}{5 + 4\cos\theta}\, d\theta$. (10%)

9. Evaluate the surface integral $\iint\limits_{S} G(\mathbf{r})dA$ for the following data. (15%)
$$G = 3xy, \quad S : z = xy, \quad 0 \le x \le 1, \quad 0 \le y \le 1$$