

國立高雄大學 102 學年度研究所碩士班招生考試試題

科目：基礎數學
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

- (1) (10%) Find the derivative of $\sqrt{x^{1/2} + x^{1/3}} \sin x$.
- (2) (10%) Evaluate the integral $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x^2 - 2xy) \exp\{-Q\} dx dy dz$ where $Q = 3x^2 + 2y^2 + z^2 + 2xy$.
- (3) (15%) Evaluate $\lim_{x \rightarrow 0} \frac{\int_0^x v^2 \exp\{v^2\} dv}{\int_0^x v^3 + v^2 \exp\{v^2\} dv}$.
- (4) (15%) Let the curves C_1 and C_2 are given by $(x, y) = (t^2, \sqrt{t^2 + t^3})$ and $(x, y) = (t^2, 0)$, respectively. Find the area between these two curves from $t = 0$ to $t = 2$.
- (5) (20%) Let A and B be defined by
- $$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & 1 & -3 \end{bmatrix}; B = \begin{bmatrix} 0 & 3 & 3 \\ -2 & 5 & 6 \\ -4 & -3 & -1 \end{bmatrix}.$$
- (a) Find the rank of A and B , respectively.
- (b) Show that the column space of B is a subspace of the column space of A .
- (6) (15%) Show that $A^T AB = 0$ if and only if $AB = 0$ for any matrices A and B such that the multiplications are defined.
- (7) (15%) Let A be a $k \times k$ nonsingular matrix, \vec{a} be a $k \times 1$ vector, and c be a scalar such that $c > (\vec{a})^T A (\vec{a}) \geq 0$. Show that the matrix $B = A - (\vec{a})(\vec{a})^T / c$ is nonsingular.