1. (20 %) Solve the differential equations. (i) $\ln(y^x)y' = x^2 y$; $y(2) = e^2$; (ii) $y' = \dfrac{2xy^3 - 3y}{3x - 3x^2 y^2 + 6y}$.

2. (10 %) Solve the differential equation $x^2 y'' - 3xy' + 4y = \ln(x)$.

3. (10 %) Find the Laplace transform of $f(t) = e^{-t} \displaystyle\int_0^t e^{\omega} \cos(\omega)\,d\omega$.

4. (10 %) Solve the initial value problem $y'(t) + y(t) = tH(t - 2)$; $y(0) = 0$,

   where the Heaviside (unit step) function is defined by $H(t) = \begin{cases} 1, & t > 0 \\ 0, & t \le 0 \end{cases}$.

5. (20 %) Given $\phi = 2xyz^2$ and $\vec{F} = xy\vec{i} - z\vec{j} + x^2\vec{k}$ following curve C: $x = t^2, y = 2t, z = t^3$ from t = 0 to t = 1,

   (a) what is $d\vec{r}$, please calculate (b) $\displaystyle\int_C \phi\, d\vec{r}$ and (c) $\displaystyle\int_C \vec{F} \times d\vec{r}$

6. (15 %) For a function defined in pieces by

$$f(x) = \begin{cases} 8, & for\, 0 < x < 2 \\ -8, & for\, 2 < x < 4 \end{cases}$$   where f(x) is periodic with period 4

   (a) Please plot this function and find its Fourier series.
   (b) What does the series converge to at x = 2? and at x = 3?

7. (15 %)

   (a) Consider $f(x) = e^{-at} H(t)$, please find its Fourier transforms.

   (b) Consider $G(\omega) = \dfrac{1}{4 + i(\omega - 2)}$, please find Inverse Fourier transform of $G(\omega)$.

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Fourier series: $f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty}\left( a_n \cos\dfrac{n\pi x}{p} + b_n \sin\dfrac{n\pi x}{p} \right)$,

$$a_0 = \dfrac{1}{p}\int_{-p}^{p} f(x)\,dx \qquad a_n = \dfrac{1}{p}\int_{-p}^{p} f(x)\cos\dfrac{n\pi x}{p}\,dx$$