國立聯合大學 102 學年度碩士班考試招生

電機工程研究所 入學考試試題

科　目：工程數學　　第 1 頁共 2 頁

『Show the details of your work.』

1. Solve the following differential equations：

   (a) $xy' + y = (xy)^{\frac{5}{2}}$ (7%)

   (b) $y'' + 4y' + 13y = \frac{1}{3}e^{-2t}\sin 3t$, $y(0) = 1$, $y'(0) = -2$ (8%)

2. (a) Find the inverse Laplace transform of the function $F(s) = \dfrac{b}{s^3(s+a)}$; a, b are positive real number.

   (7%)

   (b) Solve the following differential-integral equation. (Hint：Applying Laplace transform and convolution theorem) $y'(t) + y(t) - \int_0^t y(v)\sin(t-v)dv = -\sin t$, $y(0) = 1$ (8%)

3. (a) Find the fourier series in real form of the following function (9%)

   $f(x) = |3x|$, $-2 < x < 2$, $f(x+4n) = f(x)$

   (b) Find the fourier series in complex form of the following function (6%)

   $f(x) = e^{2x}$, $-\pi < x < \pi$, $f(x+2n\pi) = f(x)$

4. (a) If f(z) is analytic in a simply connected domain D, then for every simple closed path C in D,

   Show that $\oint_C f(z)dz = 0$ usi ng Green's theorem and Cauchy–Riemann Equations. (8%)

   (b) Calculate the complex line integral $\oint_c \dfrac{1}{z^2-1}dz$, using residue theorem.

   where C: (1) $|z| = \frac{1}{2}$, (2) $|z-1| = 1$, (3) $|z+1| = 1$, (4) $|z| = 2$ (1 2%)

5.(a)Suppose $T: R^2 \to R^2$ is a linear transformation and that $T(1,0)=(1,4)$, $T(1,1)=(2,5)$.

(1)What is $T(3,5)$? (6%)

(2) What is the null space of T ? (3%)

(3) What is the range of T ? (3%)

(b)Let $\begin{cases} x_{k+1} = \frac{1}{2}x_k + y_k \\ y_{k+1} = \frac{1}{16}x_k + \frac{1}{2}y_k \end{cases}$ $(k=0,1,2,......)$ and $x_0 = 0$, $y_0 = 1$, find x_n and y_n (13%)

6. Let $\vec{F} = xy\vec{i} + yz\vec{j} + xz\vec{k}$, calculate surface integral $\oiint_S \vec{F}\cdot\vec{n}dA$ usi ng gauss divergence theorem ,where S is the bounding surface (with outer unit normal $\vec{n}$) of the unit cube by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. (10%)