

國立聯合大學 102 學年度碩士班考試招生  
光電工程研究所 入學考試試題

科目： 工程數學 第 1 頁共 1 頁

1. Solve the following problems.

(1)  $y' = y(1 - 2y)$  (10%)

(2)  $y''' - 2y'' - 4y' + 8y = 5t$  (10%)

2. Find the Laplace transform for the function shown below. (10%)

$$f(t) = e^{-2t} \int_0^t e^{-t} \cos(t) dt$$

3. Find the Laplace inverse transform for the function shown below. (10%)

$$F(s) = \ln \left( \frac{s-3}{s+1} \right)$$

4. (1) Find the Fourier transform function  $H(f)$  of  $h(t)$ . ( $f_0, A$  are constants) (10%)

$$h(t) = 2f_0 A \sin C(2\pi f_0 t)$$

(2) Find the Fourier inverse transform of  $G(f)$ . (10%)

$$G(f) = H(f - 10f_0) + H(f + 10f_0)$$

5. Let scalar function  $f = 4x^2 + xy^2 + 9y^3z^2$  and vector function  $\vec{v} = xy\hat{x} + (x-y)^2\hat{y} + 2x^2yz\hat{z}$ . Find

(1)  $\nabla^2 f$  (5%)

(2)  $\text{curl}(\text{grad}(f))$  (5%)

(3)  $\nabla f \cdot \text{curl } \vec{v}$  (5%)

(4)  $\nabla \cdot (\nabla \times \vec{v})$  (5%)

6.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(1) Find the orthogonal matrices  $P$  and its  $P^{-1}$  to get orthogonally diagonalize  $A$ . (15%)

(2) Find the diagonal matrix  $D$  so that  $A$  is similar to  $D$ . (5%)