1. (30%) Consider a rigid body as shown in Fig. 1, where G is its center of gravity. There are two coordinate systems: one is the inertial coordinate system X-Y-Z with the origin at O, and the other is the body-fixed one, X_G-Y_G-Z_G, with the origin at G. The position of G with respect to the inertial coordinate system is $[g_1 \;\; g_2 \;\; g_3]^T$. Consider now a point P on the rigid body, whose coordinate with respect to the body-fixed coordinate system is $[p_1 \;\; p_2 \;\; p_3]^T$.
   (a) (5%) What is the coordinate of P with respect to the inertial coordinate system?
   (b) (10%) Now, the rigid body is rotated along Y_G axis with the angle of θ_A. What is the coordinate of P with respect to the inertial coordinate system?
   (c) (15%) Next, in addition to the rotation as done in Problem (b) above, the rigid body is also rotated along Z_G axis with the angle of θ_B. What is the coordinate of P with respect to the inertial coordinate system?

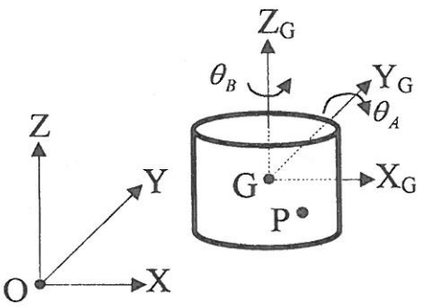

Fig. 1

2. (20%) The mass center G of a wheel of radius R is located at a distance r from its geometric center C. Initially (at $t=0$), G is directly above C, as shown in Fig. 2. Suppose that the wheel rolls without sliding at a constant angular velocity of ω.
   (a) (10%) Please find the velocity of the mass center G, as a function of time, in both X and Y directions.
   (b) (10%) Please find the acceleration of the mass center G, as a function of time, in both X and Y directions.

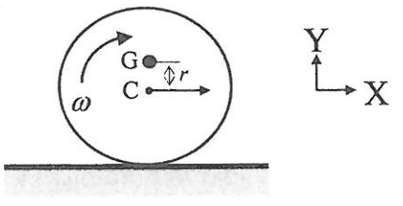

Fig. 2

3. (25%) As shown in Fig. 3, the block D moves with a speed of 3 m/s and the dimensions of the links are marked in the figure. Please answer the following questions.
   (a) (5%) Please determine the location of the instantaneous center of zero velocity.
   (b) (5%) Please calculate the velocity of the point B.
   (c) (5%) Please calculate the angular velocities of links BD and AB.
   (d) (10%) If the rod AB turns with a clockwise angular acceleration of 10 rad/sec^2 at the instant, please determine the acceleration of the point B and block D.

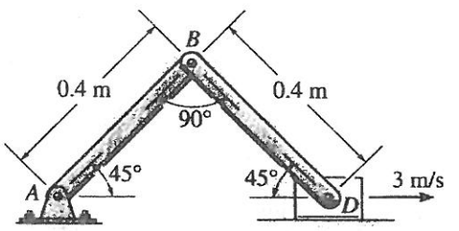

Fig. 3

4. (25%) The wheel shown in Fig. 4 weights 200N and has a radius of gyration K_G=0.18 m about its mass center G. If it is subjected to a clockwise moment of 23 Nm and rolls from rest without slipping. The spring has a stiffness of k=160 N/m and is **initially unstretched** when the moment is applied. Please answer the following question.
   (a) (5%) If the velocity of the center G is given as V_G and the angular velocity of the wheel is given as ω, please determine the relation between V_G and ω.
   (b) (5%) Please find out the kinetic energy of the wheel which can be expressed by using only the variable ω.
   (c) (15%) If the center G moves 0.15m, please determine value of the angular velocity ω of the wheel using the principle of work and energy.

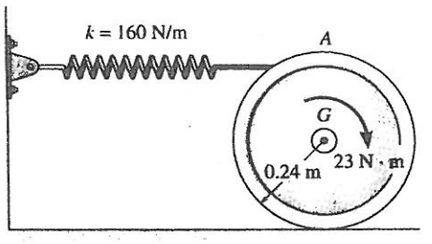

Fig. 4