國立中正大學 102 學年度碩士班招生考試試題

系所別：機械工程學系-甲組、乙組、丙組　　科目：工程數學

1. (25%) Solve the following differential equations:

(a) (10%) $\left(x^2-4y^2\right)\cdot dx+\left(2x^2+xy\right)\cdot dy=0$

(b) (15%) $x^2\cdot\dfrac{d^2y}{dx^2}+2x\cdot\dfrac{dy}{dx}=x\cdot\sin(x)+x^3\cdot e^x$

2. (25%) Laplace transformation：

(a) (5%) Derive the Laplace transforms of $\cos\omega t$ and $\sin\omega t$.

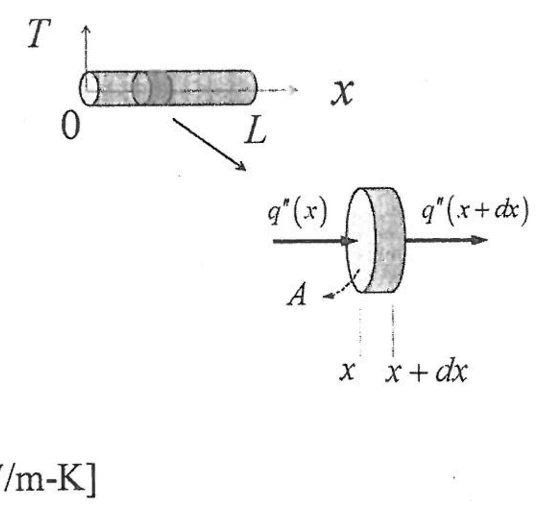
(b) (10%) Find $f(t)$ if its Laplace transform $F(s)$ equals $\dfrac{1}{s^2}\left(\dfrac{s+1}{s^2+1}\right)$.

(c) (10%) Solve the following initial value problem:

$$y''+2y'+2y=0,\ \ y(0)=1,\ \ y'(0)=-3.$$

3. (15%) One end of a metal bar is heated by a source. Assume heat transfer occurs only in the x direction. The heat conduction equation is given as:

$$q''=-k\frac{dT}{dx}$$



$T(x,t)$ = temperature [K]

$q''(x,t)$ = heat flux [W/m^2]

C_p = specific heat [J/kg-K]

ρ = density [kg/m^3]

A = cross section area [m^2]

k = heat conduction coefficient [W/m-K]

(a) (10%) Derive a partial differential equation for the temperature variation with respect to the distance x and time t.

(b) (3%) The heat source provides a constant heat flux q''_c, write this boundary condition.

(c) (2%) The temperature at $x=L$ is kept constant at T_L, write this boundary condition.

4. (10%) For a given point (2, −2, 8) on a surface $z = x^2 + y^2$, find

(a) (5%) the tangent plane to the point, and

(b) (5%) the normal line through the point.

5. (10%) Expand $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x < \pi \end{cases}$, in a Fourier series.

6. (15%) Consider a matrix $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$.

(a) (3%) Find the eigenvalues of the matrix A.

(b) (12%) Find the eigenvectors of the matrix A.