

一、單選題(共 45 分):每題有五個選項，選擇一個最適當的答案，答對一題得 3 分;未作答、答錯或多於一個選項者，該題以 0 分計算。

1. The average power of the continuous-time periodic signal  $x(t) = A \cos(\omega_0 t + \theta)$  with period  $T_0$  and  $\omega_0 = 2\pi/T_0$  is (A)  $A/2$  (B)  $A$  (C)  $A^2/2$  (D)  $A^2$  (E)  $2A^2$

2. Consider a system with the impulse response  $h(t) = e^{-\alpha t} u(t)$  ( $\alpha > 0$ ). Which one can be used to describe the basic properties of this system? (hint:  $u(t)$ : unit step function) (A) causal, memory, unstable (B) non-causal, memory, stable (C) causal, memoryless, stable (D) non-causal, memory, stable (E) causal, memory, stable

3. A linear time-invariant (LTI) system can be represented by the differential equation  $\frac{dy(t)}{dt} + ay(t) = x(t)$ . Please determine the zero-input response  $y(t)$  of

this system with the initial condition  $y(0) = 2$ . (A)  $y(t) = 2$  (B)  $y(t) = \frac{e^{-at}}{2}$

(C)  $y(t) = e^{-at}$  (D)  $y(t) = 2e^{-at}$  (E)  $y(t) = a$

4. The system defined by the input-output equation

$$y[n] = \sum_{k=-\infty}^n x[k]$$

is called the accumulator system. Which of the following is incorrect? (A) This accumulator system is a linear system. (B) This accumulator system is a system with memory. (C) This accumulator system is a time-invariant system. (D) The inverse system of the accumulator system is  $w[n] = y[n] - y[n-1]$ . (E) This accumulator system is a stable system.

5. Which of the following can be used to characterize the impulse response  $h[n]$  of an accumulator? (hint:  $u[n]$ : unit step sequence,  $\delta[n]$ : unit impulse sequence) (A)  $h[n] = \delta[n]$  (B)  $h[n] = \delta[n] + \delta[n-1]$  (C)  $h[n] = \delta[n+1] + \delta[n]$  (D)  $h[n] = u[n]$  (E)  $h[n] = u[n] - u[n-1]$

6. Which of the following discrete-time signals could not be an eigenfunction of any stable LTI system? (A)  $10^n \cdot e^{j3\omega n}$  (B)  $e^{j2\omega n} + e^{j3\omega n}$  (C)  $(1/4)^n$  (D)  $e^{j5\omega n}$  (E)  $e^{j\omega n/3}$

7. Let the discrete-time Fourier transform (DTFT) of the discrete-time sequence  $x[n]$

be  $X(e^{j\omega}) = \left( \frac{1}{1 - \alpha e^{-j\omega}} \right)^2$ . The sequence  $x[n]$  is (A)  $-\alpha^n u[-n-1]$

(B)  $\alpha^n u[n+1]$  (C)  $(n+1)\alpha^n u[n]$  (D)  $n\alpha^n u[n]$  (E)  $\alpha^n u[n]$

8. Consider the signal  $x(t) = 4 \sin(3\pi t + \frac{\pi}{4}) + 8 \cos(20\pi t + \frac{\pi}{6}) + 10 \cos(50\pi t)$ . Which

is the minimum sampling frequency? (A) 50 Hz (B)  $50\pi$  Hz (C) 20 Hz (D)  $20\pi$  Hz

(E) None of the above

9. Which of the following impulse responses corresponds to the stable LTI system? (A)

$$h[n] = n \cos\left(\frac{\pi}{4}n\right)u[n] \quad (B) \quad h[n] = 3^n u[n] \quad (C) \quad h[n] = \sum_{k=-\infty}^n \delta[k] \quad (D) \quad h[n] = 3^n u[-n+10]$$

(E) None of the above

10. Consider a signal  $x(t)$  and its Fourier transform  $X(j\omega)$ . Which of the following

corresponds to the Fourier transform of a signal  $x^*(-t)$ ? (A)  $X(j\omega)$  (B)  $X^*(j\omega)$

(C)  $X^*(-j\omega)$  (D)  $X(-j\omega)$  (E) None of the above

11. The Fourier transform of  $e^{-|t|}$  is  $2/(1+\omega^2)$ . What is the Fourier transform of

$$1/(1+t^2)? \quad (A) \quad e^{-|\omega|} \quad (B) \quad \omega e^{-|\omega|} \quad (C) \quad 2\omega e^{-|\omega|} \quad (D) \quad \pi e^{-|\omega|} \quad (E) \quad 2\pi e^{-|\omega|}$$

12. Consider a causal LTI system with frequency response  $1/(3+j\omega)$ . The system output  $y(t)$  is  $[e^{-3t} - e^{-4t}]u(t)$  for some input signal  $x(t)$ . Which of the following is the input signal  $x(t)$ ? (A)  $e^{3t}u(t)$  (B)  $e^{4t}u(t)$  (C)  $e^{-4t}u(t)$  (D)  $e^{-3t}u(t)$  (E) None of the above

13. Which of the following is not the sufficient condition for convergence of Fourier transform? (A) The signal  $x(t)$  is integrable. (B) The signal  $x(t)$  is absolutely integrable. (C) In any finite time interval, the number of maxima and minima is finite. (D) The signal  $x(t)$  has only a finite number of discontinuities within any finite time interval. (E) None of the above

14. Which of the following is correct? (A) Periodic signal must be an energy signal. (B) A memoryless system must be a causal system. (C) A memoryless system must be a non-causal system. (D) A memory system must be a causal system. (E) Periodic signal must be a causal signal.

15. Which of the following is incorrect for the periodic impulse train

$$\delta_{T_0}(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_0) \quad \text{with period } T_0? \quad (\text{hint: } \delta(t) \text{ :unit impulse function})$$

(A) The double-sided frequency spectrum is a line spectrum with space  $\frac{2\pi}{T_0}$  rad/sec between adjacent spectral lines. (B) The height of the double-sided

frequency spectrum is  $\frac{1}{T_0}$ . (C) Its Fourier series is  $\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \frac{2}{T_0} e^{-j\frac{2\pi kt}{T_0}}$ . (D) All

the Fourier series coefficients of the impulse train are identical. (E) The Fourier series coefficients of the impulse train are real valued and even.

二、計算題(共 55 分):

1. (18%) Assume that  $h_1(t)$ ,  $h_2(t)$ ,  $h_3(t)$ ,  $h_4(t)$ , and  $h_5(t)$  are the impulse responses of the five linear time-invariant (LTI) subsystems. A new system is created by these five LTI subsystems shown in Figure 1.

(A). (4%) Express the impulse response of the new system  $h(t)$  by using  $h_1(t)$ ,  $h_2(t)$ ,  $h_3(t)$ ,  $h_4(t)$ , and  $h_5(t)$ .

(B). (14%) (1). (8%) Given  $h_1(t) = e^{-t}u(t)$ ,  $h_2(t) = e^{-t}u(t)$ ,  $h_3(t) = e^{-2t}u(t)$ ,

$h_4(t) = 2e^{-t}u(t)$ , and  $h_5(t) = e^{-2t}u(t)$ , find the impulse response

$h(t)$  of the new systems.

(2). (3%) Is this system stable?

(3). (3%) Is this system causal?

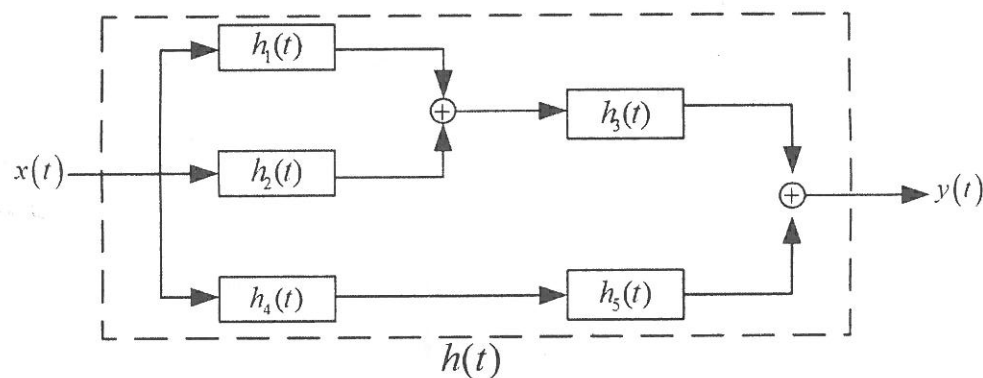


Figure 1.

2. (10%) A causal and stable LTI system has the frequency response

$$H(j\omega) = \frac{j\omega + 2}{3 - \omega^2 + 4j\omega}$$

(A). (3%) Determine a differential equation relating the input  $x(t)$  and output  $y(t)$  of the system.

(B). (3%) Determine the impulse response  $h(t)$  of the system.

(C). (4%) What is the output of the system when the input is  $x(t) = e^{-t}u(t)$ .

3. (12%) We are given the following facts about a sequence  $x[n]$

(A).  $x[n]$  is periodic with period  $N = 8$ . (B).  $\sum_{n=0}^7 x[n] = 8$ . (C).  $\sum_{n=1}^8 (-1)^n x[n] = 1$ .

(D).  $x[n]$  has the minimum power per period among the set of signals satisfying the preceding three conditions.

Determine the sequence  $x[n]$ .

4. (15%) Consider a system with impulse response  $h[n]$  for which the z-transform  $H(z)$  has the pole-zero plot shown in Figure 2.
- (A). (5%) How many possible regions of the convergence (ROCs) have that are consistent with the nature of the sequence?
- (B). (4%) Determine the ROC of  $H(z)$  if it is known that the Fourier transform exists. For this case, determine the corresponding sequence.
- (C). (3%) Is it possible for the pole-zero plot in Figure 2 to be associated with a sequence that is both stable and causal? If so, determine the sequence and its corresponding ROC.
- (D). (3%) How many possible two-sided sequences have the pole-zero plot shown in Figure?

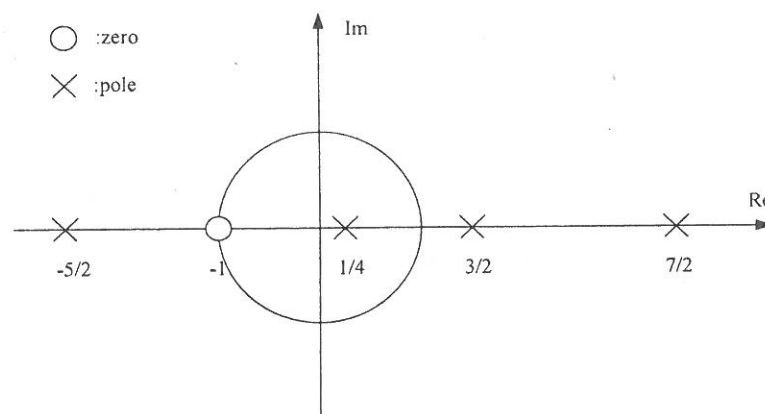


Figure 2.