

國立中正大學 102 學年度碩士班招生考試試題

電磁晶片組

系所別：電機工程學系-計算機工程組

科目：線性代數與微分方程

電力與電能處理甲組、乙組

第 2 節

第 1 頁，共 2 頁

Show all your work and write your answers clearly.

1. (10%) Solve the initial-value problem

$$y'' + y = 8 \cos 2x - 4 \sin x, \quad y(\pi/2) = -1, \quad y'(\pi/2) = 0.$$

2. (10%) Find two independent power series solutions of

$$y'' - xy = 0.$$

3. (10%) Use the Laplace transform to solve the initial-value problem

$$y'' + 16y = \delta(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 1.$$

4. (10%) Solve the initial-value problem

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 1/t \\ 1/t \end{pmatrix}, \quad \mathbf{X}(1) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

5. (10%) Solve the boundary-value problem

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(1) + y'(1) = 0.$$

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第 7 頁，共 7 頁

6. The 3×3 symmetry matrix A has 3 eigenvalues $\{0, 1, 2\}$ and corresponding eigenvectors

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

- (a) (5 %) Find A where each entry is integer.
(b) (10 %) Find the LU factorization of A .
7. Let $B = \{[1 \ 0 \ 0]^T, [0 \ 1 \ 0]^T, [0 \ 0 \ 1]^T\}$ and $B' = \{[1 \ 1 \ 0]^T, [1 \ 0 \ 1]^T, [0 \ 1 \ 1]^T\}$ be bases for \mathbb{R}^3 , and let

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

be the matrix for $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ relative to B .

- (a) (5 %) Find the transition matrix P from B' to B .
(b) (5 %) Use the matrices M and P to find $[T(v)]_B$, where $[v]_{B'} = [1 \ 0 \ -1]^T$.
(c) (5 %) Find P^{-1} .
(d) (5 %) Find the matrix M' for T relative to B' .
(e) (5 %) Find $[T(v)]_{B'}$.

8. An augmented matrix for a linear system can be expressed as

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 2 & 3 & 7 & 5 \\ 1 & 3 & a & b \end{array} \right]$$

- (a) (5 %) Find a and b for the inconsistent system.
(b) (5 %) Find a and b for the system with infinitely many solutions.