國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：統計學【經濟所碩士班】　　題號：403002

※本科目依簡章規定「不可以」使用計算機　　

請依序橫寫，在答案卷作答. 5題共100分.

1. (20%)
The amount of gasolines sold daily at a service station is uniformly distributed with a minimum of 2,000 gallons and a maximum of 5,000 gallons.
(a). Find the probability that daily sales will fall between 2,500 and 3,000 gallons.
(b). What is the probability that the service station will sell at least 4,000 gallons ?

2. (20%)
Given the independent random variables X_1, X_2 and X_3 with the probability density

$$f_1(x_1) = \begin{cases} e^{-x_1} & for\ x_1 > 0 \\ 0 & elsewhere \end{cases},$$

$$f_2(x_2) = \begin{cases} 2e^{-2x_2} & for\ x_2 > 0 \\ 0 & elsewhere \end{cases},$$

$$f_3(x_3) = \begin{cases} 3e^{-3x_3} & for\ x_3 > 0 \\ 0 & elsewhere \end{cases}.$$

Find the probability $P(X_1 + X_2 \leq 1, X_3 > 1)$.

3. (20%)
Given a random sample of size N from a population having the density

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)} & for\ x > \theta \\ 0 & elsewhere \end{cases}.$$

Find the maximum likelihood estimator for the parameter θ.

4. (20%)
Suppose the weight X_1, height X_2, and age X_3 of a randomly chosen male have a multivariate normal distribution with means 170, 68, and 40 and variances 400, 16, and 256, and with covariance $Cov(X_1, X_2) = 64$, $Cov(X_1, X_3) = 128$, and $Cov(X_2, X_3) = 0$. Find the conditional expectation of X_1 given $X_2 = 72$ and $X_3 = 24$, i.e., $E(X_1 \mid X_2 = 72, X_3 = 24) = ?$

5. (20%)
A paint manufacturer wants to determine the average drying time of a new interior wall paint. If for 12 test areas of equal size he obtained an average drying time of 66.3 minutes and a standard deviation of 8.4 minutes. Construct a 95% confidence interval for the true mean μ.
(Note that if Z is a standard normal, $Prob(Z \leq z) = 0.95$ when $z = 1.64$, and $Prob(Z \leq z) = 0.975$ when $z = 1.96$. If U has a student t distribution with k degree of freedom, then $Prob(U \leq t) = 0.975$ when $t = 2.201$ and $k = 11$. $Prob(U \leq t) = 0.975$ when $t = 2.179$ and $k = 12$.)