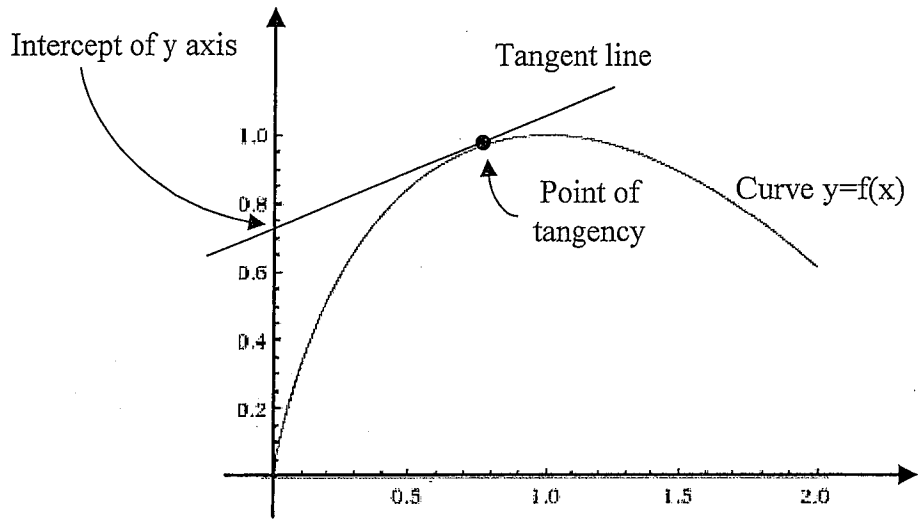
國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【海工系碩士班甲組】　　題號：459007

※本科目依簡章規定「不可以」使用計算機　　

1. [Ordinary Differential Equations] **(15%)**
A given curve $y = f(x)$ passes through the point (1, 1). The intercept of y axis for all the tangent lines of the curve are equal to the x coordinates of the points of tangency. Find the equation of the curve.



2. [Laplace Transform] **(10%)**
Use **two** different methods to find the inverse transformation $\mathcal{L}^{-1}[\frac{1}{s^4-4s^2}]$

3. [Linear Algebra] **(10%)**
Find the values of k so that the vectors $[1 \quad -2 \quad 3 \quad 2]$, $[2 \quad k+1 \quad 6 \quad 8]$, and $[-1 \quad 3 \quad 2k-1 \quad -1]$ are linearly independent.

4. [Vector Calculus] **(15%)**
Given the motion on the curve $\mathbf{r}(t) = \cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$, find the points (coordinates) with maximum speed and acceleration. Also find the tangential acceleration $\mathbf{a}_t$ of the motion.

5. [Fourier Series] **(15%)**
Find the Fourier series for a given function $f(x)$ with intervals specified:
$f(x) = x, \ -\pi < x < \pi,$
$f(x + 2k\pi) = f(x), \ -\infty < x < \infty$ and $k = \pm$integer.

6. [Residue Integration] **(10%)**
Evaluate the improper integral $\int_0^\infty \frac{dx}{1+x^4}$
with four simple poles at $z_1 = e^{\frac{\pi i}{4}}$, $z_2 = e^{\frac{3\pi i}{4}}$, $z_3 = e^{\frac{-3\pi i}{4}}$, $z_4 = e^{\frac{-\pi i}{4}}$ on a full circle.

7. [Partial Differential Equation] **(25%)**
(a) What are the names of the three types of PDE and a typical field of application in each type? (5%)
(b) Solve the following PDE using the Method of Separation of Variables: (20%)
$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$, for $0 \le x \le L, \ t > 0;$
for the motion of a string released from rest, subject to
boundary conditions $u(0,t) = 0, \ u(L,t) = 0, \ t > 0;$ and
initial conditions $u(x,0) = f(x), \ \frac{\partial u(x,0)}{\partial t} = g(x) = 0, \ 0 < x < L.$