

# 國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：微積分【公事所碩士班甲組選考】

題號：444005

※本科目依簡章規定「不可以」使用計算機

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請依題號順序作答，違者扣分。

1. Find the derivatives for the following functions. 20%

(a)  $f(z) = \sqrt{z}e^{-z}$       (b)  $f(x) = \ln(3x^2 + 5)$

(c) Find  $dy/dx$  where  $x \cos y + y \sin x = 1$

(d) Find  $dy/dx$  where  $y = u(1-u)^3$  and  $u = \frac{1}{3x-2}$

2. Let  $f$  be a function with  $f(x) > 0$  for all  $x$ . Set  $g = 1/f$ . 15%

(a) If  $f$  is increasing in an interval around  $x_0$ , what about  $g$ ?

(b) If  $f$  has a local maximum at  $x_1$ , what about  $g$ ?

(c) If  $f$  is concave down at  $x_2$ , what about  $g$ ?

3. Evaluate the following. 40 %

(a)  $\int x\sqrt{x+1} dx$     (b)  $\int x^2 \ln x dx$     (c)  $\int_1^{\infty} \frac{dx}{6x+1}$     (d)  $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$

(e)  $\int_0^1 x f''(x) dx$  where  $f$  is twice differentiable with  $f(0)=5$ ,  $f(1)=4$ , and  $f'(1) = 1$ .

(f)  $\int_1^{\infty} 3x^{-p} dx$ , indicate the value of  $p$  so that the integral diverges.

(g)  $\int_0^2 \int_0^x e^{x^2} dy dx$ . and sketch the region of integration.

(h) Determine if the following sequences converge

a.  $\sum_{n=1}^{\infty} \frac{1}{n^2+3}$     b.  $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$     c.  $\sum_{n=1}^{\infty} \frac{1}{ne^n}$

4. By looking at their Taylor series, decide which of the following functions is the largest and which is the smallest, for small positive  $x$ . 15%

(a)  $\cos x$     (b)  $\frac{1}{1-x^2}$     (c)  $1 + \sin x$

5. Find the mean age in a country, using the age density function given by  
 $p(t) = 0.0015, 0 \leq t \leq 40$ ;  $p(t) = 0.0262 - 0.00028t, 40 < t \leq 93.3$ ; 10%

(A mean of a distribution density function  $f$  is defined as  $\int_{-\infty}^{\infty} xf(x) dx$ )

