國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：微積分丙【企管系甲班碩士班丙組選考】　　題號：441005

※本科目依簡章規定「不可以」使用計算機　　

請依題號順序作答，違者扣分。

1. Find the derivatives for the following functions. 20%

i. $f(u)=\sqrt{u}e^{-u}$

ii. $f(x)=\ln(x^2+\pi)$

iii. Find dy/dx where $x\sin y+y\cos x=1$

iv. Find dy/dx where $y=u(1-u)^3$ and $u=\dfrac{1}{3x-2}$

2. Sketch the function $f(x)=x^3-3x^2;-1\le x\le 3$. 15 %

Indicate

i. the critical points,

ii. the reflection point(s),

iii. the global maxima and minima,

iv. the increasing and decreasing and concavity of the function

3. Evaluate the following. 40 %

i. $\int x^2\ln x\,dx$　ii. $\int_1^{\infty}\dfrac{dx}{5x+1}$　iii. $\int x\sqrt{x+1}\,dx$　iv. $\lim_{x\to 0}\dfrac{e^x-x-1}{x^2}$

v. $\int_0^1 xf''(x)dx$ where f is twice differentiable with $f(0)=5$, $f(1)=4$, and $f'(1)=1$.

vi. $\int_1^{\infty}5x^{-p}dx$, indicate the value of p so that the integral diverges.

vii. $\int_0^2\int_0^x e^{x^2}dydx$. and sketch the region of integration.

viii. Determine if the following sequences converge

a. $\sum_{n=1}^{\infty}\cos(\frac{1}{n})$　b. $\sum_{n=1}^{\infty}\dfrac{1}{n^2+3}$　c. $\sum_{n=1}^{\infty}\dfrac{1}{ne^n}$

4. By looking at their Taylor series, decide which of the following functions is the largest and which is the smallest, for small positive x. 15%

i. $1+\sin x$　ii. $\cos x$　iii. $\dfrac{1}{1-x^2}$

5. Find the median age in a country, using the age density function given by

$p(t)=0.0015, 0\le t\le 40$; $p(t)=0.0262-0.00028t, 40<t\le 93.3$; 10%

(A median of a distribution density function f is a value T such that $\int_{-\infty}^{T}f(x)dx=0.5$)