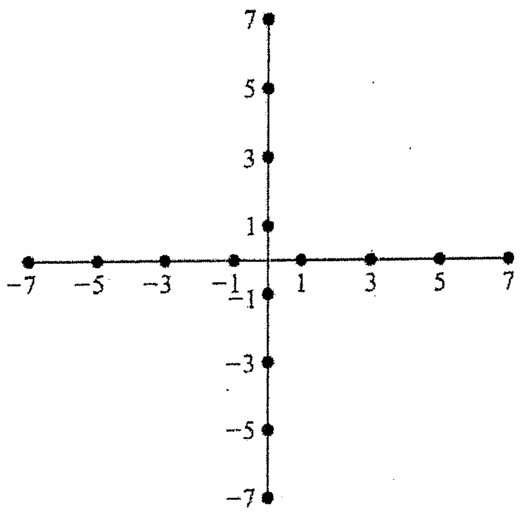
國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：通訊理論【通訊所碩士班甲組】　　　　　　題號：437005
※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　　共2頁第1頁

1. [20] Signal Constellation and Decision Boundaries

For the QAM signal constellation shown in the following figure, determine the optimum decision boundaries for the detector, assuming that the signal to noise ratio (SNR) is sufficiently high so that errors only occur between adjacent points.



2. [20] Random Process

Please prove that if the input to a stable linear time-invariant filter is a wide-sense stationary random process, then the output of the filter is also wide-sense stationary.

3. [20] Characteristic Function and Gaussian Random Variable

The characteristic function of a random variable X is defined as the statistical average: $E(e^{jvX}) \equiv \psi(jvX) = \int_{-\infty}^{\infty} e^{jvx} p(x)dx$.

A. [10] Find the characteristic function of the Gaussian random variable.

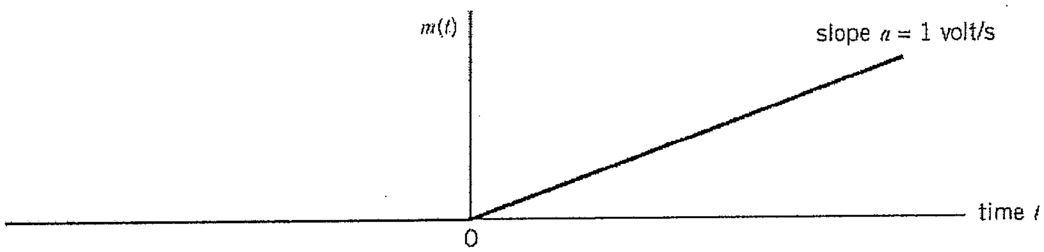
B. [10] Show that the sum Y of N independent and identically distributed (i.i.d.) Gaussian random variables, X_i, $i = 1,2,...,N$, is a Gaussian random variable.

4. [20] Phase/Frequency Modulation

For the message signal $m(t)$ shown in the following figure:

A. [10] Please draw a diagram to illustrate the phase modulated wave.

B. [10] Please draw a diagram to illustrate the frequency modulated wave.



5. [20] Fourier Transform:

A. [10] The Fourier transform of a decaying exponential pulse is given by: $\exp(-at)u(t) \rightleftharpoons \dfrac{1}{a+j2\pi f}$, $a > 0$, where $u(t)=\begin{cases}1, & t>0\\ \frac{1}{2}, & t=0\\ 0, & t<0\end{cases}$. Please show that the Fourier transform of a double exponential pulse is given by: $\exp(-a|t|) \rightleftharpoons \dfrac{2a}{a^2+(2\pi f)^2}$, $a>0$.

B. [5] Please show that the Fourier transform of a signum function is given by $\operatorname{sgn}(t) \rightleftharpoons \dfrac{1}{j\pi f}$, where $\operatorname{sgn}(t)=\begin{cases}+1, & t>0\\ 0, & t=0\\ -1, & t<0\end{cases}$.

C. [5] Please find the Fourier transform of a unit step function $u(t)$.

Property	Mathematical Description
Linearity	$ag_1(t)+bg_2(t) \rightleftharpoons aG_1(f)+bG_2(f)$, where a and b are constants.
Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$, where a is a constant.
Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$.
Time shifting	$g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$.
Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f-f_c)$.
Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$.
Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$.
Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$.
Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$.
Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G(f-\lambda)\,d\lambda$.
Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$.
Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \lvert g(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2 df$.