國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】　　題號：437002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

1. (20 %) For each of the following statements, mark "**O**" if the statement is TRUE and "**X**" if the statement is FALSE. You do NOT need to provide any justification.

   (a) (　　). The fixed points **u** of a transformation $T(\mathbf{u}) = \mathbf{Bu}$ are eigenvectors with its eigenvalue being one.

   (b) (　　). The set of all vectors of the form $(3a+b+1, 2a, b)$ is a subspace of $\mathbb{R}^3$.

   (c) (　　). The dimension of Span$\{e^t, e^{3t}, 2e^t + 3e^{3t}, e^t - 2e^{3t}\}$ is 4.

   (d) (　　). The eigenvalues of a square matrix must be distinct.

   (e) (　　). Let two vectors be $\mathbf{u} = \begin{bmatrix} i \\ 6 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2i \\ 0 \\ 1 \end{bmatrix}$. Then, $\mathbf{u} \perp \mathbf{v}$.

2. (10%) Let **A** and **B** be 3x3 matrices with det(**A**) = 5, det(**B**) = 10 and det(**A** + **B**) = 60. Decide the following values.

   (a) det(**A** + **A**).

   (b) det($\mathbf{A}^2\mathbf{B} + \mathbf{AB}^2$).

3. (20%) Define $T : P_2 \to \mathbb{R}^3$ by $T(p) = \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$ where $p(t)$ in P_2 can be expressed as $p(t) = at^2 + bt + c$.

   (a) Find the image under T when $p(t) = 6 + 2t$.

   (b) Show that T is a linear transformation.

   (c) Find the kernel of T.

   (d) Find the matrix for T relative to the basis $\{1, t, t^2\}$ for P_2. (This means that the matrix will act on the coordinates of p).

4. (10%) A square matrix is called upper triangular if all the entries below the main diagonal are zero. The product of upper triangular matrices is

   (a) lower triangular matrix,

   (b) upper triangular matrix,

   (c) diagonal matrix.

   Please select the best answer and you do NOT need to provide any justification.

5. (10%) Which of the following is not a linear equation of (x_1, x_2, x_3)?

   (a) $x_1 + 4x_2 + 1 = x_3$

   (b) $x_1 = 1$

   (c) $x_1 + 4x_2 - \sqrt{2}x_3 = \sqrt{4}$

   (d) $x_1 + 4x_1x_2 - \sqrt{2}x_3 = \sqrt{4}$

   Please select the best answer and you do NOT need to provide any justification.

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6. (10%) If $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then the eigenvalues of $\mathbf{A}$ is

(a) $(1, 2, -1))$.

(b) $(1, 1/2, -1)$.

(c) $(1, -2, -1)$.

(d) $(1, -1/2, -1)$.

Please select the best answer and you do NOT need to provide any justification.

7. (10%) If $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ -2 & -4 & -9 \end{bmatrix}$ and $\det(\mathbf{A}) = 0$, then $\text{rank}(\mathbf{A})$ is

(a) 0.

(b) 1.

(c) 2.

(d) 3.

Please select the best answer and you do NOT need to provide any justification.

8. (10%) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ be four different vectors in $\mathbb{R}^3$. Then

(a) they must be linearly independent.

(b) they must be linearly dependent.

(c) they must be either linearly independent or linearly dependent.

(d) none of the above hold.

Please select the best answer and you do NOT need to provide any justification.