國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【材光系碩士班乙組】 題號：439001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘） 共1頁第1頁

1. Solve $(2y^2-9xy)dx+(3xy-6x^2)dy=0$ (15%)

2. Evaluate $\oint_C x^2y\,dx-xy^2\,dy$, where C is the boundary of the region (15%)
$x^2+y^2\le 4,\quad x\ge 0,\quad y\ge 0$

3. The Legendre polynomial as a polynomial solution of the Legendre equation
$(1-x^2)y''-2xy'+n(n+1)y=0$, where $n = 0, 1, 2,$
From $(1-2xr+r^2)^{-1/2}=\sum_{n=0}^{\infty}P_n(x)r^n$, calculate $\int_{-1}^{1}[P_n(x)]^2dx$ (20%)

4. (a). Find the eigenvalues, eigenfunctions and verify the orthogonality of the obtained eigenfunctions for the following initial value problem of the differential equation.
$y''+8y'+(\lambda+16)y=0,\ y(0)=0,\ y(\pi)=0$ (15%)

(b). Based on the answer of (a), determine the Fourier coefficient for $f(x)=e^{4x}$ and $p(x)=1$. (10%)

5. Find the temperature $u(x,t)$ of one − dimensional heat equation, $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ (c is a constant), for an adiabatic bar (length = L) satisfying $u(x,0)=f(x)$. (15%)

6. Find the Laplace integral, $\int_0^{\infty}\frac{\cos\omega x}{k^2+\omega^2}d\omega=\frac{\pi e^{-kx}}{2k}$, through the Fourier cosine integral of $f(x)=e^{-kx}$, where $x>0$ and $k>0$ (show the details of your work). (10%)