

# 國立中山大學 102 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【光電所碩士班】

題號：435001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）

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## Question 1 (20%)

(1) Prove following vector calculations. (10%)

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a}, \mathbf{b})^2$$

(2) Calculate the distance between two lines,  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{4}$  and  $x = y = z$ . (10%)

## Question 2 (20%)

Calculate the following integration using the Laplace transform. Note that even if you obtain an answer, you do not receive any score unless you use the Laplace transform.

$$\int_0^{\infty} \frac{\sin ax}{x} dx$$

## Question 3 (20%)

The Fourier transform is often used to relate the time domain parameter  $t$  and the frequency domain parameter  $\omega$ . Suppose that a function  $x(t)$  has Fourier transform  $X(j\omega)$ . Now, consider another function  $g(t)$  whose shape is the same as the shape of  $X(j\omega)$ , that is

$$g(t) = X(jt).$$

Show that the Fourier transform  $G(j\omega)$  of  $g(t)$  has the same shape of  $2\pi x(-t)$ , that is, show that

$$G(j\omega) = 2\pi x(-\omega).$$

## Question 4 (20%)

Solve following differential equations to obtain general solutions.

(1)  $\cos x \sin y \frac{dy}{dx} = \sin x \cos y$  (6%)

(2)  $(x + y + 1) \frac{dy}{dx} = x + y - 1$  (6%)

(3)  $y + \frac{dy}{dx} = xy^3$  (8%)

## Question 5 (20%)

Assuming  $z = re^{i\theta}$  and  $\zeta = \rho e^{i\phi}$ , calculate following values.

(1)  $\operatorname{Re} \frac{z + \zeta}{z - \zeta}$  (10%)

(2)  $\operatorname{Im} \frac{z + \zeta}{z - \zeta}$  (10%)

(END OF QUESTIONS)

