

系所組別： 土木工程學系甲、乙、丁組

考試科目： 工程數學

考試日期：0225，節次：3

1. (20%) Solve the following differential equations:

(a) $\frac{dy}{dx} + p(x)y = r(x), y(x_0) = y_0.$

(b) $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = x$

2. (20%) Solve the following system of differential equations

$$\frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \\ z(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

3. (20%) Consider the eigenvalue problem

$$x^2 y'' + xy' + \lambda y = 0, \quad (1 < x < a)$$

$$y(1) = 0, \quad y(a) = 0.$$

Show that the eigenvalues and eigenfunctions are

$$\lambda_n = \frac{n^2 \pi^2}{(\ln a)^2}, \quad \phi_n(x) = \sin\left(n\pi \frac{\ln x}{\ln a}\right)$$

For $n = 1, 2, \dots$

4. (20%) Solve the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < l, t > 0,$$

which satisfies the boundary conditions $u(0, t) = 0, u(l, t) = 0$, and the initial conditions

$$u(x, 0) = \sin\left(\frac{2\pi x}{l}\right), \quad \frac{\partial u(x, 0)}{\partial t} = 0.$$

5. (20%) show that the Cauchy principal value

$$P \int_{-\infty}^{\infty} \frac{e^{itx}}{x\pi i} dx = \begin{cases} 1 & (t > 0), \\ 0 & (t = 0), \\ -1 & (t < 0). \end{cases}$$