

系所組別： 工程科學系甲、乙、丙、丁、戊、己組

考試科目： 工程數學

考試日期：0225，節次：3

1. Solve $\frac{d^3 y}{dt^3} + 3\frac{d^2 y}{dt^2} + 3\frac{dy}{dt} + y = e^{-t}$ with the initial conditions

$$\frac{d^2 y}{dt^2}(0) = \frac{dy}{dt}(0) = y(0) = 1. \quad (20\%)$$

2. Evaluate $\iint \bar{F} \cdot \bar{n} d\sigma$ across the part of sphere $x^2 + y^2 + z^2 = 4$ lying between

$$z = 1 \text{ and } z = 2, \text{ where } \bar{F} = x\bar{i} + y\bar{j} + z\bar{k}. \quad (20\%)$$

3. (a) Use n linearly independent vectors $\bar{u}_1 \sim \bar{u}_n$ to generate n orthonormal

vectors $\bar{e}_1 \sim \bar{e}_n$. What are these vectors $\bar{e}_1 \sim \bar{e}_n$? (8%)

(b) Find these $\alpha_1 \sim \alpha_n$ for $\bar{y} = \sum_{i=1}^n \alpha_i \bar{e}_i$, where \bar{y} is a given vector. (5%)

4. (a) Find the value $(1+i)^{2-i}$, (10%) (b) Calculate $\sin\left(\frac{\pi}{2} + i \ln 2\right)$ (5%)

© Calculate $\int_{-\infty}^{\infty} \frac{x^2 \cos(mx)}{(x^2 + a^2)(x^2 + b^2)} dx$, $a > b > 0$. (12%)

5. Solve $\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}\right)u = 0$, $a \leq r \leq \infty$, $0 \leq \theta \leq 2\pi$ with the boundary condition $u(a, \theta) = f(\theta)$. Hint: u is a single-value function of θ and finite at $r = \infty$. (20%)