1. (5%) Given the hash function $h(x) = x \bmod 13$, show how the sequence 784, 281, 1141, 18, 1, 329, 620, 43, 31, 684 would be inserted in the order given in initially empty array of integer indexed 0 to 12. (If there are collisions, describe your choice to solve the collisions.)

2. (5%)Is the relation $\{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (3,4), (4,3)\}$ an equivalence equation on $\{1, 2, 3, 4\}$? How many (distinct) equivalence classes are there? List the equivalence classes.

3. (10%)Given the relations: $R_1 = \{(1,x), (2,x),(2,y),(3,y)\}$, $R_2 = \{(x,a), (x,b), (y,a), (y,c)\}$.
   (1)Derive the matrix A_1 of the relation R_1 relative to the orderings: $1,2,3; x,y$.
   (2) Derive the matrix A_2 of the relation R_2 relative to the orderings: $x, y; a, b, c$.
   (3) Derive the matrix product A_1A_2
   (4) How do you interpret the product in (3)?

4. Given the sequence $\{t_n\}$ defined by the equations: $t_1 = t_2 = t_3 = 1$,
   $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 4$.
   (1) (2%)Derive t_4 and t_5.
   (2) (4%)Design a recursive algorithm to compute t_n, $n \geq 1$.
   (3) (4%)Prove that your algorithm is correct using mathematical induction.

5. (1)(5%) A Trojan horse scan (THS) program is used to detect Trojan horse (TH) programs in a computer. At one computer maintenance shop, approximately 15% of the to-be scanned computers have the TH programs. Within those that have the TH programs, THS program reports approximately 95 percent positive. Among those that do not have TH programs, THS reports approximately 2 percent positive. Find the probability that a to-be scanned computer has the TH program if the THS program reports positive.

   (2)(5%)Given gcd(196, 425) = 1, find the inverse s of 196 modulo 425 satisfying $0 < s < 425$.

6. (10%)Solve the recurrence relation: $a_n = 3a_{n-1} + 10\, a_{n-2}$; $a_0=4$, $a_1 = 13$.

7. Given the adjacency matrix of a graph as follows:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	1	0	0	0	1	0
v_2	1	0	1	1	0	1	1
v_3	0	1	0	1	0	0	0
v_4	0	1	1	0	1	0	0
v_5	0	0	0	1	0	1	1
v_6	1	1	0	0	1	0	1
v_7	0	1	0	0	1	1	0

(1) (5%)Describe whether the path ($v_2, v_3, v_4, v_2, v_6, v_1, v_2$) in the graph is a simple path, a cycle, a simple cycle or none of these.

(2) (5%)Is there an Euler cycle in the graph? Explain.

(3) (5%)Does the graph have a Hamilton cycle? Explain.

(4) (5%)If A is the adjacency matrix of the graph, what does the entry in row v_2 and column v_3 of A^3 represent?

8. (10%) Some binary tree has a pre-order traversal as ABFGCDE, and a post-order traversal as GFBEDCA.

(1)Draw the binary tree and list the order in which the vertices are processed using inorder traversal.

(2) Use topological sort to sort the nodes, starting from the root.

9. (1) (5%)Prove that in any Boolean algebra, $(x\,(x + y \cdot 0))' = x'$ for all x and y.

(2) (5%) Design a logic circuit with three inputs that outputs 1 precisely when two or three inputs have value 1, using only NOR gates.

10. (10%) Design a finite-state automaton M that accepts exactly the strings generated by the regular grammar G_1. The production rules of G_1 are:
$\alpha \rightarrow y\alpha$, $\alpha \rightarrow xN$, $N \rightarrow yN$, $N \rightarrow y$. The starting symbol is α. The set of terminal symbols and the set of nonterminal symbol are $\{x,y\}$ and $\{\alpha, N\}$, respectively.