

系所組別： 水利及海洋工程學系甲、乙組

考試科目： 工程數學

考試日期：0225 · 節次：3

1. (10%) Determine the following statement is **true** or **false**.

(1) (1%) Given an invertible matrix \mathbf{A} , $\det(\mathbf{A}^{-1}) = 1/\det(\mathbf{A})$, where \det denotes determinant.

(2) (1%) The series $\sum_{m=0}^{\infty} \frac{x^m}{m!}$ always converges.

(3) (1%) $\det(\mathbf{A} + \mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B})$.

(4) (1%) Given a 2×2 matrix \mathbf{A} with determinant of 3, $\det(2\mathbf{A}) = 6$.

(5) (1%) $f * 1 = f$, where $*$ denotes convolution operator.

(6) (1%) $\text{div}(\text{curl } \mathbf{v}) = 0$, where \mathbf{v} is a vector.

(7) (1%) Both \mathbf{A} and \mathbf{B} are $n \times n$ matrices, $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + 2\mathbf{A}\mathbf{B} + \mathbf{B}^2$.

(8) (1%) A matrix and its transpose do not have the same rank.

(9) (1%) Given an invertible matrix \mathbf{A} , $(\mathbf{A}^2)^{-1} = (\mathbf{A}^{-1})^2$

(10) (1%) The determinant of an orthogonal matrix has value of 0.

2. (15%) Given $f(t) = \begin{cases} c, & 0 < t < 1 \\ h(t), & t > 1 \end{cases}$, where c is a constant and $h(t)$ is a function. If the Laplace transform of $f(t)$ is $F(s)$. Find the Laplace transform of $f(t+1)$ in terms of $F(s)$.

3. (15%) Suppose that a cylindrical tank has diameter D . The top of the tank is open and a hole with diameter d at the bottom. (1) (10%) Find the height of water (h) in the tank at any time if the initial height of the water when the hole is opened is H . (2) (5%) When will the tank be empty?

4. (15%) Let \mathbf{A} be an $n \times n$ matrix with eigenvalues $\lambda_1, \dots, \lambda_n$. Prove that (1) (5%) \mathbf{A}^{-1} has eigenvalues $1/\lambda_1, \dots, 1/\lambda_n$; (2) (5%) \mathbf{A}^m ($m = 1, 2, \dots$) has eigenvalues $\lambda_1^m, \dots, \lambda_n^m$; (3) (5%) $\mathbf{A} - k\mathbf{I}$ has eigenvalues $\lambda_1 - k, \dots, \lambda_n - k$.

5. (15%) Solve the initial value problem, $x^2 y'' - 3xy' + 3y = 2x^4 e^x$, $y(1) = -2$, $y'(1) = -8 + 2e$.

6. (15%) Given a cardioid $r = a(1 - \cos\theta)$, where $a > 0$ is a constant and $0 \leq \theta \leq 2\pi$.

(1) (8%) Find the area of this cardioid; (2) (7%) Find the perimeter of this cardioid.

7. (15%) Find the Fourier series of the function $f(x) = x^2$, $-\pi < x < \pi$, which is assumed to have the period of 2π .